



**A DECISION SUPPORT TOOL FOR THRIFT  
SAVINGS PLAN INVESTORS**

THESIS

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AFIT/GEM/ENV/04M-02

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THESIS

Presented to the Faculty

Department of Systems and Engineering Management

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the  
Degree of Master of Science in Engineering Management

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March 2004

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## **Abstract**

The Thrift Savings Plan is the defined contribution retirement plan for federal government employees. It is one of the largest retirement plans in the United States. The plan offers five different investment options: two funds concentrate on government securities and corporate bonds, two funds span the United States stock market, and one fund focuses on international stocks. These funds give investors the opportunity to diversify among a wide range of securities.

This thesis examines the funds offered by the plan and creates a portfolio selection tool that uses investor inputs. The tool uses three mathematical models: optimization, based on Markowitz's (1952) Modern Portfolio Theory; simulation, based on the combination of optimization results and investor profiles; and selection, based on the simulation results and investor preferences. Results are presented for four investors. Other investors can use these results to gain insight, but the greatest benefit is derived from individual implementation. The tool requires only a few user inputs, and it can be operated without any external assistance. The research has further found that one investment option should not be part of any portfolio, and some portfolios can be risky to individuals for differing reasons.

## Acknowledgments

Thanks to my wife.

Christopher J. Blanchette

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# A DECISION SUPPORT TOOL FOR THRIFT SAVINGS PLAN INVESTORS

## Chapter 1: Introduction

### 1.1 General Background

According to the Federal Reserve Board's 2001 Survey of Consumer Finances, 59.2% of United States households save (Federal Reserve Bulletin, 2003:5). People save for a number of reasons. Many save for specific purchases such as furniture, a car, or a house. Some save for unforeseen contingencies, while others save because the action of saving makes them feel good (Browning and Lusardi, 1996:1797). The most popular reason for saving in the U.S. is retirement, which is the primary purpose of the Thrift Savings Plan. Over 32% of Americans who save cite retirement as their top reason, and 28.4% of their household assets are invested in specific retirement accounts (Federal Reserve Bulletin, 2003:6, 9).

After establishing their financial goals, individuals face enormous challenges when deciding upon an investment strategy. Should they invest in high return holdings? Should they invest in holdings that guarantee a specific return? Should their decisions be based on their goals, time horizons, risk aversion, or possibly a combination of all these factors? Are there other factors, such as ethical standards, to consider when choosing an investment? Investment companies provide insight into which vehicles may be most appropriate. Income sources such as money markets, government bonds, and tax-free municipal bonds offer a low risk/low return alternative. These vehicles are good for short-term investments and investors with high risk aversion. Growth funds are recommended for investors with low risk aversion and long time horizons, and growth and income funds attempt to achieve balance between the two extremes. Investment managers often recommend people build a portfolio that incorporates all investment

vehicles, noting that specific funds are available for income, tax-free income, growth and income, and growth (Fisher and Statman, 1997:15-17).

Given the range of questions and choices individuals face, investment advice is available from a number of sources. Direct investment advice can come from mutual fund companies and independent investment managers. Private businesses offer advice to their employees regarding company retirement plans either directly or through management companies. Magazines and periodicals are widely available, from peer-reviewed journals such as *The Journal of Finance*, to store shelf magazines such as *Money*. Online companies, such as The Motley Fool and Yahoo!, offer advice and pool information sources. Software packages offer simulation and selection programs. These programs are examples of decision support tools designed to assist individuals in specific investment scenarios.

Decision support systems are intended to help a business or individual collect and analyze data to make decisions (Post and Anderson, 2003:296). These systems belong to the field of operations research, which is a methodology for providing a quantitative basis for decisions (Morse and Kimball, 1970:1). Decision support systems originated during World War II when the military needed to allocate scarce resources to support various operations (Ragsdale, 2001:12). These systems proved to be invaluable to the war effort, but the development and construction of computerized systems was quite expensive and cost prohibitive in the private sector. However, the increasing use of computers in the workplace during the 1960s brought an expansion in the use of decision support systems by businesses (Power, 2003:n. pag.). The development of desktop personal computers has extended the growth of these systems into the household. With this proliferation of decision support systems, individuals often fail to recognize their use in everyday activities. They can be as complex as expert systems, with

thousands of equations designed to support decisions for global organizations, or as simple as an Excel spreadsheet designed for personal budgets (Post and Anderson, 2003:296).

Although their capabilities may vary, decision support systems share one common task: to evaluate alternatives and assist in selecting the best course of action (Ragsdale, 2001:1). Thus, they are a useful tool for individuals making investment decisions. In fact, savvy investors can perform their own research and analysis with existing decision support tools or create personalized models. However, many individuals considering investments have neither the required background nor the desire for this level of sophisticated analysis. This is particularly true of employees choosing to invest in their company's respective retirement plans. Federal employee participation in the government's retirement program is a prime example and the subject of this research effort.

## **1.2 Specific Background**

The United States Congress established the Thrift Savings Plan (TSP) with passage of the Federal Employees' Retirement System Act of 1986 to provide retirement income to federal civilian employees. The TSP is a defined contribution retirement savings and investment plan. This means the retirement income received depends on the amount contributed and the earnings on those contributions. Analogous to the 401(k) plans offered by many private companies, employee contributions and earnings on those contributions are tax-deferred until withdrawal (Federal Retirement Thrift Investment Board, 2001:2). Although the TSP was initially intended for federal civilian employees, enrollment was extended to members of the uniformed services in calendar year 2002.

The TSP offers five investment funds: the Government Securities Investment (G) Fund, the Fixed Income Index Investment (F) Fund, the Common Stock Index Investment (C) Fund, the Small Capitalization Stock Index Investment (S) Fund, and the International Stock Index Investment (I) Fund (Federal Retirement Thrift Investment Board, 2001). Contributions to the TSP are made each pay period, which is bi-weekly for civilians and monthly for military members. Civilians can contribute a whole dollar amount or whole percentage of their pay, while military members can contribute whole percentages of base, incentive, special, and bonus pay. The contribution can be allocated between the five funds in any whole percentage. Existing TSP balances can be redistributed at any time through the use of inter-fund transfers. The historical annual return over the life of each fund varies from 6.8% to 13.5%, with the monthly standard deviation of each fund ranging from about 20% of the expected return to nearly 10 times the expected return.

### **1.3 Research Problem**

Although there are pamphlets and web pages that offer basic information about the TSP program, there is minimal advice concerning how individuals should allocate their TSP contributions and holdings. Even those with substantial investment knowledge may have difficulty creating their ideal portfolios. Therefore, the goal of the decision support system being developed during this research effort is to help solve the fundamental problem faced by all TSP investors: How should TSP investments be allocated to best match investor objectives?

The proposed solution to this problem will consist of three mathematical models: a nonlinear program for optimization, a simulation of investor cash flows, and a goal program for alternative selection. The optimization model will create a set of alternative investments (i.e.,

portfolios) for TSP investors. A portfolio will be considered optimized when it minimizes risk for a specified level of return. Given this assumption, the measure for risk must be determined for this model to be implemented. Chapter 2 will discuss the various risk measures that can be used. Given a set of portfolio choices, the simulation model will incorporate an investor's inputs to create probability distributions of the future value, measured in dollars, of each investment alternative. Finally, the goal program will assist an investor in selecting the most appropriate alternative based on the distributions of the portfolios' returns. After these specific mathematical models are developed, the resulting decision tool will be used to address the following questions.

- What is the optimal set of TSP portfolios (the alternative set) for an individual?
- What is the future value of various investment streams for the identified alternatives?
- Which alternative best matches an individual's goals?

As part of this research, a literature review will be accomplished to investigate the appropriateness of these models and specifically answer the following questions.

- What are the accepted methods for portfolio optimization based on various risk/return scenarios? Which method is the most appropriate for this optimization problem?
- Is simulation an appropriate model for investment decisions? Can probability distributions of returns be utilized to assist investors?
- What goals are important to investors in selecting specific portfolios? How should the goals be used during investment decisions?
- What are the commonly used models for portfolio allocation? How do they compare to the proposed method of alternative optimization and selection?

## **1.4 Methodology**

Once the applicability of portfolio optimization and simulation is confirmed, this research will focus on creating a specific tool for TSP investors. The data used for this research represents a population sample, and the first challenge is deciding how to interpret the data to represent the population. Individual data points are based on a several time horizons, and the population can be represented by many different distributions. Calculations, such as mean, variance, and covariance, and statistical tests will be used to find the most appropriate data representation and population estimates. These estimates will be the primary inputs for the optimization, simulation, and selection models. The expected risk and return of the optimized portfolios will be based specifically on the sample data given for the individual funds. The simulation will use investor inputs in conjunction with the optimized portfolio data to estimate probability distributions of final dollar values for individual investment streams. Finally, the selection model will combine investor goals with simulation results to assist investors in choosing a specific portfolio.

## **1.5 Scope and Limitations**

The scope of this research is to build a decision tool that TSP investors can use to assist in their allocation decision. The tool must be simple enough to be operated by each investor, since it is unknown whether the government is willing to offer direct investment advice. The decision tool created by this research must be dynamic, meaning that the recommendations will change as time passes. The optimized set of portfolios will likely change as new data becomes available, and the simulation and decision models should change as account balances and investor goals change. TSP investors should not consider investment allocations to be a one time

decision; they must continue to update allocations.

The scope is limited in the fact that it will focus only on the TSP and the investment opportunities available to federal employees. However, the research findings will apply to more than just individual investor decisions. Portfolio selection can be more than stock and bond selection, and other portfolio models (i.e. product selection, property and equipment selection, or even capital budgeting methods) will also benefit from this research. While the goals and alternatives may change, the structure for selection remains the same, but the structure also serves as a limitation. The support tool will be based on specific optimization, simulation, and selection methodologies. Those who disagree with the selected methodologies will not be inclined to utilize this tool. Some investors may not use the decision tool even if these methodologies produce desirable results. They may have an aversion to results from computer products, or they may not agree with the theories behind operations research techniques. However, even those investors who do not use the tool may gain insight into the importance of their TSP allocation decision.

## **1.6 Review of Chapters**

Chapter 2 consists of a literature review and provides a background on retirement plans, specifically the TSP. It also covers the various applications of decision analysis techniques, concentrating on investment decision models. Commonly used investment models are discussed as well. Chapter 3 covers the various methodologies used in the research, and justifies their use. Specifically, it details the development of the three mathematical models used for this decision support system. Chapter 4 documents the tests of the support tool as it is used to evaluate investment options and select the optimal portfolio(s) for sample individuals. Chapter 5

discusses the results and conclusions based on the research and demonstrates the usefulness of the techniques for other research problems. Finally, Chapter 5 presents the applicability of the decision system for future use.

## Chapter 2: Literature Review

### 2.1 Employee Retirement Plans

Employee retirement plans can take two forms: defined benefit plans and defined contribution plans. The Department of Labor's Bureau of Labor Statistics states that defined benefit plans use "a specific predetermined formula to calculate the amount of an employee's future benefit" (U.S. Department of Labor, 2003:n. pag.). The benefit formula is typically based on an employee's earnings and years of service. Contributions to defined benefit plans generally come from the employer, although employee contributions are possible (IRS: Choosing a Retirement Plan, 2004:n. pag.). The Department of Labor's Bureau of Labor Statistics describes a defined contribution plan as follows.

*A defined contribution plan is a type of retirement plan in which the amount of the employer's annual contribution is specified. Individual accounts are set up for participants, and benefits are based on the amounts credited to these accounts (through employer contributions and, if applicable, employee contributions), plus any investment earnings on the money in the account (U.S. Department of Labor, 2003:n. pag.).*

The amount contributed is specified by the employer, the employee, or both depending on how the plan is formatted. The only contribution limitations are those specified by the Internal Revenue Service (IRS). The types of defined contribution plans include 401(k) plans, 403(b) plans, 457 plans, profit-sharing plans, thrift plans, retirement savings plans, stock bonus plans, employee stock ownership plans (ESOPs), and money purchase pension plans. The most popular of these is the 401(k).

## 2.2 401(k) Plans

The term 401(k) refers to the provision of the Revenue Act of 1978, titled Internal Revenue Code (IRC) Section 401(k). This act was the first permanent provision authorizing the use of salary reductions as a source of retirement income (McDonnell, 2000:1). A 401(k) plan is a retirement plan in which employees can choose to contribute part of their compensation to the plan on a tax-deferred basis. In other words, the employee's contribution to the 401(k) plan is not subject to income tax until it is withdrawn. The contribution is, however, subject to Social Security, Medicare, and federal unemployment tax at the time it is earned (IRS: Topic 424 – 401(k) Plans, 2004:n. pag.). The tax-deferred advantages have made 401(k) plans very popular; in fact, they have become the fastest growing retirement plan in the U.S.. At the end of 2000, there were over 327,000 different 401(k) plans, with over 42 million participants (McDonnell, 2000:1).

Earnings on the contributions are tax-deferred as well, and these earnings are subject to income tax only. Contributions to 401(k) plans may be invested in stocks, bonds, guaranteed investment contracts (GICs), and cash equivalents. The allocation decision is up to the individual contributor, subject to the limitations of the asset classes the employer makes available. Distributions from a 401(k) plan may be made at age 59 ½ without penalty. However, early withdrawal of funds is subject to a 10% penalty unless the withdrawal is for an IRS approved hardship.

401(k) programs are employer sponsored, meaning that individuals can contribute only if their employers have an established 401(k) program. Furthermore, contribution amounts are limited by the IRS. The regular contribution limit for 2004 is \$13,000, and the additional contribution limit for participants age 50 and older is \$3,000. These amounts increase annually

by \$1,000 until 2007, when contribution limits will be indexed to inflation. In addition to an employee's participation in a 401(k) program, employers have the option of making contributions on behalf of the employee. Employers may choose to contribute a guaranteed dollar amount, provide matching contributions (often with a set limit), or not contribute at all. There is no IRS limitation regarding the amount of an employer's contribution; however, the combined employer and employee contributions are limited to the lesser of 25% of the employee's compensation or \$40,000.

## **2.3 The Thrift Savings Plan**

### **2.3.1 Overview**

A 401(k) program is not available to federal employees since it was established for for-profit companies. Therefore, the United States government created the Thrift Savings Plan (TSP), which operates under the same rules as a 401(k) plan, with passage of the Federal Employees' Retirement System Act of 1986 (Federal Retirement Thrift Investment Board, 2001:2). The TSP has since grown to become the largest of all 401(k)-type programs (USDA, 2000:1). The plan is administered by the Federal Retirement Thrift Investment Board, an independent government agency with just over 100 employees and a five member Board of Directors. The Board's sole purpose is to manage the TSP in the best interests of the plan's participants and their beneficiaries (Federal Retirement Thrift Investment Board, 2001:2).

The TSP is available to all federal government employees, although contributions are limited according to an employee's retirement system classification. Civilian employees under the Federal Employee's Retirement System (FERS) may contribute up to 14% of their base pay. The employees' agencies will automatically contribute 1% and match employee contributions

dollar for dollar up to 3% and fifty cents to the dollar from 3-5%, with no agency matching beyond 5%. A breakdown of the employee and employer contributions up to 5% is outlined in Table 2.1 below.

**Table 2.1 Employee and Employer TSP Contributions**

<b>Employee</b>	<b>Automatic</b>	<b>Matching</b>	<b>Total</b>
0%	1%	0%	1%
1%	1%	1%	3%
2%	1%	2%	5%
3%	1%	3%	7%
4%	1%	3.5%	8.5%
5%	1%	4%	10%

Employees who fall under the Civil Service Retirement System (CSRS) can presently contribute up to 9% of their base pay, but they do not receive matching contributions from their agency since they also qualify for a defined benefit retirement plan. Military members may contribute up to 9% of their basic pay and 100% of any other pay. Additionally, military agencies may choose to authorize matching contributions for critically manned career fields. Maximum base pay contributions for all civilian and military employees will increase by 1% in 2005. In 2006, the only restraint will be the IRS limitation on annual 401(k) contributions.

Regardless of employment classification, TSP contributions can be allocated between the five available funds in any whole percentage. Furthermore, allocation adjustments can be made at any time, and existing TSP balances can be redistributed at any time through the use of inter-fund transfers. Finally, all TSP participants enjoy the same benefits as 401(k) participants (Federal Retirement Thrift Investment Board, 2001:5).

### **2.3.2 TSP Investment Options**

The TSP offers five investment funds: the Government Securities Investment (G) Fund, the Fixed Income Index Investment (F) Fund, the Common Stock Index Investment (C) Fund, the Small Capitalization Stock Index Investment (S) Fund, and the International Stock Index Investment (I) Fund. Each fund is invested in different areas of the financial market. The G and F funds are fixed income funds, meaning the interest rate earned on the securities in these funds is set in advance. The C, S, and I funds are stock funds giving TSP investors the opportunity to invest in three broad stock market indices. Earnings in the G Fund come from interest paid by the U.S. government on short-term securities specifically issued to the TSP. The other four funds derive their earnings from interest paid by the U.S. government and U.S. corporations, capital gains related to the value of securities, and dividends paid by U.S. corporations. The F, C, S, and I Funds use a passive investment strategy, meaning the fund managers replicate the index being tracked, instead of attempting to outperform it. As with all investment funds, earnings are reduced by administrative expenses to cover management fees and trading costs. The F, C, S, and I funds attempt to follow a common index, although actual returns vary due to the previously mentioned expenses and the fact that the funds cannot follow the indices exactly (Federal Retirement Thrift Investment Board, 2001:18-19). Fund returns for the past 10 years are summarized in Figure 2.1, and specific information regarding each fund is provided below. The S Fund and I Fund are relatively new, having been introduced in May 2001. Therefore the return data in Table 2.2 for these funds prior to May 2001 is represented by the returns of the indices they track.

**Table 2.2 TSP Fund Returns (Thrift Savings Plan for Federal Employees, 2004:n. pag.)**

**10-Year Summary of TSP Annual Returns**

<b>Year</b>	<b>G Fund %</b>	<b>F Fund %</b>	<b>C Fund %</b>	<b>S Fund %</b>	<b>I Fund %</b>
1994	7.22	-2.96	1.33	-2.66	7.75
1995	7.03	18.31	37.41	33.48	11.27
1996	6.76	3.66	22.85	17.18	6.14
1997	6.77	9.60	33.17	25.68	1.55
1998	5.74	8.70	28.44	8.63	20.09
1999	5.99	-0.85	20.95	35.49	26.72
2000	6.42	11.67	-9.14	-15.77	-14.17
2001	5.39	8.61	-11.94	-9.04	-21.94
2002	5.00	10.27	-22.05	-18.14	-15.98
2003	4.11	4.11	28.54	42.92	37.94
<b>10-Year Compound</b>	6.04	6.95	10.99	9.70	4.32

2.3.2.1 G Fund. The G Fund is comprised of investments in short-term U.S. Treasury securities. The fund guarantees a return equal to the average return on outstanding U.S. Treasury securities with 4 or more years to maturity. While the return fluctuates annually, the G Fund is risk-free in terms of credit risk. Credit risk is the risk that a bond issuer may refuse to pay interest or principal. The U.S. government backs all securities in the G Fund, so principal and interest will always be paid. The average annual rate of return of the G Fund since its inception in 1988 is approximately 6.74% (Federal Retirement Thrift Investment Board, 2001:19-20).

2.3.2.2 F Fund. The F Fund invests in Barclays U.S. Debt Index Fund, which tracks the Lehman Brothers U.S. Aggregate (LBA) index. This index measures the performance of the U.S. bond market, which is comprised of a combination of government, corporate, and mortgage-backed securities. It provides an opportunity for a higher rate of return than the G

Fund, but it also has slightly more risk. The corporate bond portion of the fund is subject to credit risk. The fund is also subject to market risk, which is the risk that the market value of the fund will change as interest rates change. Finally, there is pre-payment risk, which occurs when mortgage-backed securities and corporate bonds are paid early. The F Fund has enjoyed an average annual return of 8.05% since its inception in 1988. The risk associated with the fund is evident when annual returns are examined. For instance, the return was -2.96% in 1994 (Federal Retirement Thrift Investment Board, 2001:20-22).

2.3.2.3 C Fund. The C Fund invests in Barclays Equity Index Fund, which tracks the Standard & Poor's 500 (S&P 500) stock index. The S&P 500 is an index of 500 companies traded in U.S. markets. These companies represent over 100 different industries in four major business sectors. Since the C Fund uses a passive investment strategy, stock purchases are made only to replicate the S&P 500 Index. The risk and return of the C Fund is significantly higher than either of the two bond funds. Since its inception in 1988, annual returns have ranged from a high of 37.41% in 1995 to a low of -22.05% in 2002 (Federal Retirement Thrift Investment Board, 2001:23-24).

2.3.2.4 S Fund. The S Fund invests in Barclays Extended Market Index Fund, which is comprised of medium and small companies represented by the Wilshire 4500 stock index. This index represents those stocks that are not in the S&P 500. The total number of companies in the index numbers over 6,000. Since this is a stock fund, it is more risky than the bond funds. The S fund also tends to be more volatile than the C Fund, since it invests primarily in small and medium companies. Annual return of the fund has varied from -18.14% in 2002 to 42.92% in 1999 (Federal Retirement Thrift Investment Board, 2001:24-26).

2.3.2.5 I Fund. The I Fund is an international index fund. It invests in Barclays Europe, Australasia, and Far East (EAFE) Index Fund. This fund tracks the Morgan Stanley Capital International EAFE stock index. This index follows the performance of several major foreign corporations. Not only does this fund carry the same risk as the other stock funds, but it also carries the risk of currency fluctuations since the value of the index rises and falls with the value of the U.S. dollar. The I Fund return for 2003 was 37.94%, but the 2001 return (I fund and EAFE index combined) was -21.94% (Federal Retirement Thrift Investment Board, 2001:26-27).

## **2.4 Decision Analysis**

The Federal Retirement Thrift Investment Board makes information about the funds widely available to participants. Additionally, the board provides information regarding loans, transfers, survivor benefits, withdrawals, and tax advantages. However, the board does not recommend specific funds or combinations of funds. This decision is left to the employee. While it is understandable why the board does not make allocation recommendations, it is also a major drawback. Many TSP participants do not have the financial background needed to make this decision, yet their retirement depends greatly on their allocation decisions.

### **2.4.1 History of Decision Support**

Decision support systems originated during World War II (WWII) when the military needed to allocate scarce resources to support various operations (Ragsdale, 2001:12). In the early years after WWII, building the computer-based information systems required for decision support was expensive, so the advancement of decision support systems was limited. This began to change in 1965 with the introduction of the IBM System 360 and other mainframe computer

systems. These computers finally made it practical to develop management information systems (MIS) in companies large enough to afford the technology. The first MIS outputs were periodic reports for functions such as accounting and sales (Power, 2003:n. pag.).

Decision support systems became more commonplace in the corporate world in the late 1960s because of the efforts of pioneers such as Peter Keen and Charles Stabell. The continuing evolution of decision support systems was evidenced by the first publication of peer-reviewed articles in 1968 and the first book on the subject, *Management Decision Systems: Computer-Based Support for Decision Making*, by Michael S. Scott Morton, in 1971. By the late 1970s, decision support systems were gaining recognition as tools capable of helping decision makers at all levels of an organization, although computer capability limited the pace of growth (Power, 2003:n. pag.).

Advancements in the 1980s included the development of software tools to support decision-making and the introduction of artificial intelligence for expert decision systems. By 1990, the use of data warehousing and online analytical processing greatly increased the amount of data available to decision support systems. In the early 1990s, these systems moved from mainframe computers to desktops. Today we can access web-based decision support systems from our home computers (Power: 2003:n. pag.).

#### **2.4.2 Mathematical Models**

Decision support systems often take advantage of computer-based mathematical models to analyze real-world situations (Ragsdale, 2001). Most mathematical models fall into one of three categories: prescriptive, predictive, and descriptive. Each type of model contains both dependent and independent variables. The relationship between these types of variables may or may not be known, and the values of the independent variables may or may not be known. The

uncertainty associated with both the values of the independent variables and the relationship between the dependent and independent variables determines the category of a model.

In a prescriptive model, the relationship between the dependent and independent variables is well defined, and the values of the independent variables are under the decision maker's control. The challenge is finding the values for the independent variables that give the most desirable result for the dependent variables. A prescriptive model helps guide a decision maker by providing a recommended course of action. Asset allocation in an investment portfolio is an example of a prescriptive model. The decision to maximize return without exceeding a defined risk level (dependent variables) could be made by selecting among different investment choices (independent variables).

Sometimes the relationship between the dependent and independent variables is not well defined, even though the values of the independent variables are known. Models that attempt to determine the ensuing relationship are known as predictive models. The variables used to make the prediction are called the independent variables, and the variables being predicted are the dependent variables. Regression and forecasting are typical predictive models. A commonly used example is the relationship between the various characteristics of a house (square footage, number of rooms, size, location, etc.) and its value.

In some situations, the relationship between the dependent and independent variables is known, but the values of the independent variables are uncertain. Models that attempt to describe the final outcome of the dependent variable based on the known relationship and unknown independent variable values are known as descriptive models. Simulations and inventory models are common examples of descriptive models (Ragsdale, 2001:6-8). The proposed TSP model will utilize simulation to estimate the future values of investor cash flows.

## **2.5 Portfolio Optimization Theories**

There are several theories dedicated to the practice of portfolio optimization. These models attempt to balance risk and return. The most common theories include the Markowitz model, capital asset pricing model, and the arbitrage pricing theory. A brief introduction to risk and these theories are discussed in the following sections. In terms of TSP applicability, this research has found that the Markowitz model is most appropriate. After reviewing each of these theories, some of the general characteristics associated with other portfolio models will be briefly discussed.

### **2.5.1 Risk**

Risk can be defined in a number of ways. Webster's defines it generically as exposure to the chance of injury or loss (Braham, 1993:573). It However, financial risk is typically evaluated in terms of the uncertainty of future returns (Reilly and Brown, 2003:5). For instance, Markowitz (1952) defines risk as standard deviation of return (variance). This measure works well when the return distribution is symmetrical. Other measures consider semivariance (returns below expectations), and include deviation below the mean, returns below zero, and returns below a benchmark (Reilly and Brown, 2003:211-212). Philippe Artzner and others relate risk to the variability of future values instead of the variability of returns (Artzner and others, 1999:205).

### **2.5.2 The Markowitz Model**

Awarded the Nobel Prize in Economics in 1990 for his seminal research, Markowitz was the first researcher to analyze the overall risk of a portfolio of securities instead of the individual risk of single securities. Using overall portfolio variance as a direct measure of risk, he showed that combinations of securities lie on the efficient frontier in terms of overall risk and return

(Markowitz, 1952). A portfolio is considered to be efficient when it has the lowest standard deviation for a specific level of return or the highest return for a specific level of standard deviation. The efficient frontier is the ensuing collection of all efficient portfolios (Reilly and Brown, 2003:211).

Markowitz' research is commonly referred to as Modern Portfolio Theory (MPT). Within this theory, the efficient frontier is established based on the following assumptions (Reilly and Brown, 2003:211).

- 1. Investors consider each investment alternative as being represented by a probability distribution of expected returns over some holding period.*
- 2. Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.*
- 3. Investors estimate the risk of the portfolio on the basis of variability of expected returns.*
- 4. Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance of returns only.*
- 5. For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk.*

The expected return for each portfolio along the efficient frontier is the weighted average of the expected returns of the individual securities (Markowitz, 1957: 81). This can be expressed as

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (2.1)$$

where  $E(r_p)$  is the expected return of the portfolio,  $E(r_i)$  is the expected return of security  $i$ ,  $w_i$  is the weight of security  $i$  in the portfolio, and  $n$  is the total number of securities. The risk of an individual security is measured by

$$\sigma = \sqrt{\sum_{i=1}^n \frac{(r_i - E(r_i))^2}{n}} \quad (2.2)$$

where  $\sigma$  is the standard deviation,  $r_i$  is the return of the security for data point  $i$ , and  $n$  is the total number of data points (Reilly and Brown, 2003:213). Equation 2.2 is the standard deviation of a population; for a sample, the formula would use  $(n-1)$  in place of  $n$  in the denominator.

Equation 2.2 calculates the risk of an individual security; however, the risk of a portfolio of securities also depends on the interaction between the securities. Markowitz uses a measure known as covariance to calculate this interaction. A positive covariance indicates the security returns move in the same direction (relative to their means), while a negative covariance indicates the returns move in opposite directions. The covariance between two securities,  $i$  and  $j$ , is (Markowitz, 1957: 80):

$$Cov_{ij} = \sum_{k=1}^n \frac{(r_{ik} - E(r_i))(r_{jk} - E(r_j))}{n} \quad (2.3)$$

where  $Cov_{ij}$  is the covariance between securities  $i$  and  $j$ ,  $r_{ik}$  is the return of security  $i$  at data point  $k$ , and  $r_{jk}$  is the return of security  $j$  at data point  $k$ . Equation 2.3 quantifies the benefits of diversification. If investors put their money in securities that all lose value at the same time

(high positive covariance), then they are not as diversified as they assumed (Fabozzi, Gupta, and Markowitz, 2002:8). However, if they put their money in investments with low positive covariance (or even negative covariance), the probability of losing money is greatly reduced.

For a portfolio of securities, the standard deviation is the weighted sum of the covariances between the individual securities (Markowitz, 1957:81). This relationship is expressed as

$$\sigma_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i w_j Cov_{ij}} \quad (2.4)$$

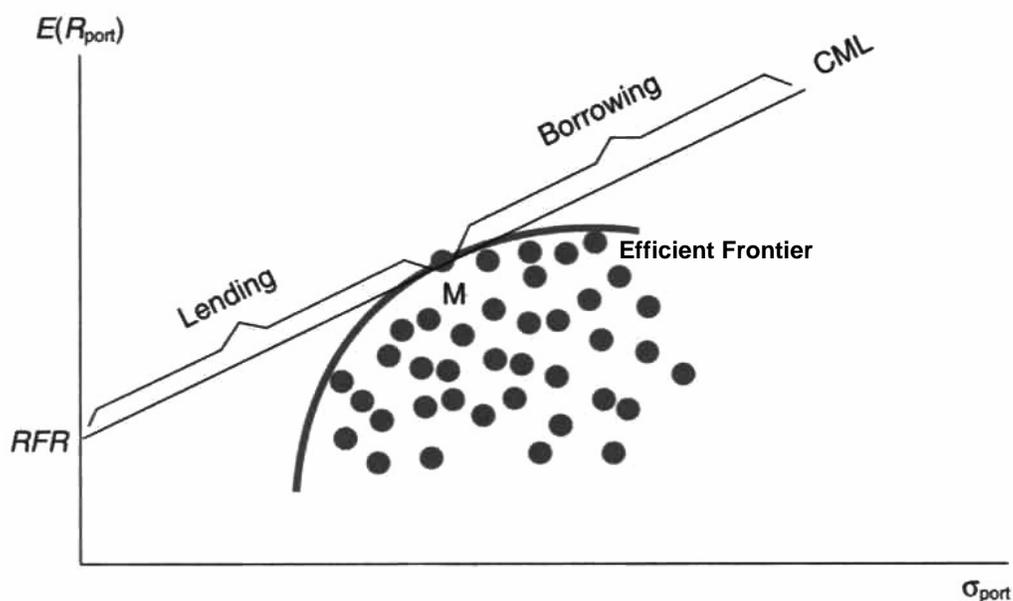
where  $\sigma_p$  is the portfolio standard deviation and  $n$  is the total number of securities. This equation incorporates the variance of individual securities since the covariance between a security and itself is its variance. Equations 2.1 and 2.4 provide the tools necessary to solve for the efficient frontier defined earlier.

Modern Portfolio Theory identifies an efficient frontier for investors to choose specific portfolios. Although MPT does not offer advice pertaining to which portfolio is most appropriate, the model does offer insight into the tradeoff between risk and return. The data required for applying this model to the TSP is readily available, and additional models could be used to assist in the selection between portfolios.

### **2.5.3 The Capital Asset Pricing Model**

The capital asset pricing model (CAPM) was developed in three separate articles by Sharpe (1964), Litmer (1965), and Mossin (1966). A textbook allocation methodology taught to many business students, the CAPM creates a set of predictions for returns on risky assets (Bodie, Kane, and Marcus, 2002:264). This theorem uses Markowitz's Modern Portfolio Theory with

the addition of a risk-free asset. A risk-free asset is defined as any asset with a known future return. This risk-free return is usually lower than the expected return on a risky asset, in which investors expect to receive a premium due to the uncertainty of future returns. CAPM states that more risk-averse investors should hold more of their investments in risk-free assets. It also states that the composition of risky assets should be the same for all investors. For example, if the model recommends risky assets should be 60% bonds and 40% stocks, then this ratio should remain constant regardless of the total percentage of dollars dedicated to risky assets (Canner and Mankiw, 1997:181). The combination of risky assets defined by the model is known as the market portfolio. The market portfolio occurs on the tangency line drawn from the risk-free asset to the efficient frontier and is known as the capital market line (CML). Figure 2.1 below displays how the market portfolio and CML is derived.



**Figure 2.1 The Efficient Frontier and the Capital Market Line (Reilly and Brown, 2003:243)**

In Figure 2.1, the x-axis is portfolio standard deviation and the y-axis is expected return. The curved line is the efficient frontier, as calculated by modern portfolio theory. Point M (i.e. the point of tangency) represents the market portfolio. In an optimal world, all investors will hold this market portfolio since they are all using Markowitz's analysis. RFR (risk free return) represents the return of the risk free asset, and the line from RFR running tangent to M represents various combinations of the market portfolio and the risk-free asset. This line is the capital market line. The CML to the left of M represents where investors lend at the risk-free rate, and the CML to the right of this point represents optimal portfolios where investors borrow at the risk-free rate to purchase more risky assets. As shown in the figure, these combinations are always superior to the efficient frontier of risky assets except the point at which an investor's portfolio is composed entirely of risky assets (Bodie, Kane, and Marcus, 2002:265-266). The equations for expected return and standard deviation for CAPM portfolios is as follows:

$$E(R_p) = w_{RF} * (RFR) + (1 - w_{RF}) * E(R_R) \quad (2.5)$$

$$\sigma_p = (1 - w_{RF}) * \sigma_R \quad (2.6)$$

where  $E(R_p)$  is the expected portfolio return,  $w_{RF}$  is the weight of the investment in the risk free asset,  $RFR$  is the risk free return,  $E(R_R)$  is the expected return on the risky assets,  $\sigma_p$  is the portfolio's standard deviation, and  $\sigma_R$  is the standard deviation of the risky assets.

If the market is in equilibrium, all assets will fall on the CAPM tangency line since their returns will match those required. If the future returns of assets can be estimated, the assets may fall on, above, or below the line. Assets falling below the line are overvalued since their expected return is lower than what is required based on their risk. Assets above the line are

considered undervalued since their returns are higher than what their risk levels demand (Reilly and Brown, 2003:248-251). A wise investor will choose stocks above the tangency line. However, finding and buying undervalued assets would require a much more active form of portfolio management and is beyond the goals of this research.

The major purpose of CAPM is to determine the required rate of return for a specific risky asset. It is based on a variable called Beta. A standardized measure of systematic risk, Beta ( $\beta$ ) is the risk of a specific asset related to the risk of the market. It is defined as

$$\beta_i = Cov_{i,M} / \sigma_M^2 \quad (2.7)$$

where  $Cov_{i,M}$  is the covariance of risky asset  $i$  with the market portfolio ( $M$ ) and  $\sigma_M$  is the standard deviation of the market portfolio. Assets with a  $\beta$  value greater than 1 are more risky than the market portfolio, so they would demand a higher rate of return than the market portfolio. Assets with a  $\beta$  value less than 1 are less risky, so they would demand a lower return. This return can be considered the required rate of return of an asset.

One weakness of the CAPM is it assumes the return on the risk-free asset is known and guaranteed. Most investments do not guarantee a specific return, and even the guaranteed returns on cash and certificates of deposit make them risky assets when real returns are considered (returns adjusted for inflation). While the return is constant, inflation is not. When there is not a risk-free investment, the ratio between risky assets cannot be held constant while remaining on the efficient frontier.

This weakness becomes a problem for TSP applicability since the safest investment in the TSP portfolio is the G fund. While this fund has never had a negative return, it does experience

variation. Additionally, most investors do not know the G Fund's return in advance. Therefore, the TSP does not have a fund that could be classified as a risk-free asset. Since there is no completely risk-free investment, the optimal portfolios are only those available along the efficient frontier of risky assets (Bodie, Kane, and Marcus, 2002:236). However, the Markowitz model would apply even if the G fund were risk-free. The mean return of a risk-free asset is the asset's specified return and the standard deviation is zero.

#### **2.5.4 Arbitrage Pricing Theory**

The arbitrage pricing theory (APT) expands the Markowitz and CAPM assumption of covariance as a measure of risk. From an investing standpoint, arbitrage is known as “the exploitation of a security in such a way that risk-free economic profits may be earned” (Bodie, Kane, and Marcus, 2002:320). Developed by Ross in the 1970s, the APT is based on three assumptions (Reilly and Brown, 2003:280-281):

- 1. Capital markets are perfectly competitive.*
- 2. Investors always prefer more wealth to less wealth with certainty.*
- 3. The stochastic process generating asset returns can be expressed as a linear function of a set of K risk factors (or indices).*

The APT assumes that the covariance between assets is not the only factor affecting a return. There are other possible factors such as inflation, gross domestic product, interest rate changes, and major disasters. Since each asset reacts differently to various factors, the challenge in APT is determining which factors are applicable and estimating the expected return of the asset. If a specific asset does not meet the expected return, an investor can enter into arbitrage arrangements in which overvalued assets are sold short and undervalued assets are purchased with the proceeds. A short sale is the sale of a stock that a person does not actually own, with the guarantee that the person will buy an equal number of shares at a later date. The arbitrage

process can continue until the relevant prices of the assets match their expected returns (Reilly and Brown, 2003:284-286).

The APT model has been extensively tested, with strong evidence both for and against it. Regardless of this evidence, there are some fundamental problems that make it difficult to use with the TSP. First, developing an accurate APT model for the TSP is beyond the scope of this research; it would require a great deal of investigation to determine the factors defining the relationship between risk and return. Second, the APT theoretically works well in finding pricing errors in individual securities. Investment funds that follow indices are much less likely to contain major errors prone to arbitrage, although APT can identify mutual fund risk exposures. Third, the TSP does not allow short sales of funds; therefore, investors could not profit from arbitrage. They could only profit by changing allocations to increase purchase of an undervalued fund. Finally, the APT is a model best suited for market timing. If investors find a difference between the price of an asset and its expected return, arbitrage will bring the asset value back in line with expectations (a common practice of hedge funds). If APT is used in this manner, TSP investors would need to follow the market daily in order to make accurate investment decisions. This research, on the other hand, will attempt to match investment decisions to long-term goals and expectations. It would be impractical to recommend a different set of portfolios on a daily basis.

### **2.5.5 Other Models**

Many other models attempt to extend Markowitz's efficient frontier model. One simple way is to minimize absolute deviation (absolute value of the deviation from the mean) instead of standard deviation. This type of model utilizes linear programming, whereas Markowitz's model requires non-linear programming. Other models attempt to account for situations where the

normality assumption is invalid by replacing variance with other variables such as semi-variance (deviation below the mean) or the worst quantile of the return distribution (Benati, 2003:572-573).

Perhaps the most common alternative approach is the Value at Risk (VaR) model, which measures the probability of the greatest potential loss of a portfolio for a defined period of time. For example, for a confidence level of 95% over a time horizon of one year, a VaR of \$100 would indicate the largest tolerated loss over a one-year period is \$100 with 95% probability (Tardivo, 2002:16-17). While the decision support tool in this thesis uses Markowitz's model for optimization, the logic behind the VaR risk measure is important and will be discussed again in the next chapter.

## **2.6 Portfolio Allocation Practices**

Most portfolio managers today, while believing in portfolio diversification, do not follow any of the optimization methodologies. Instead, they attempt to assess an individual investor's risk aversion, a factor the optimization theories do not directly address, and recommend specific asset allocations accordingly. For example, Fidelity Investments uses investor questionnaires for its managers to assign points which are used to guide investors to one of the company's model portfolios (Fisher, 1997:15-17).

Most mutual fund companies also recommend asset allocations for their investors. However, these asset allocation recommendations only guide investors in the selection of security classes and not in specific investments. Fidelity Investments offers four types of model portfolios, ranging from a 100% short-term portfolio to a 100% stock portfolio. Vanguard's portfolios follow a similar pattern, ranging from income to growth (Fisher, 1997:14). T. Rowe

Price has five model portfolios, ranging from a low-risk strategy with 25% in stocks, 40% in bonds, and 35% in cash; to a high-risk strategy with 100% invested in stocks (Fisher, 1997:17). Charles Schwab also has five model portfolios, ranging from conservative to aggressive. The conservative plan recommends 15% in large U.S. stocks, 5% in international stocks, 55% in bonds, and 25% in cash. The aggressive plan puts 40% in large U.S. stocks, 25% in small U.S. stocks, 30% in international stocks, and 5% in cash (Fisher, 1997:20).

Another common portfolio model is a pyramid of assets. Putnam Company follows this format; it uses income and tax-free funds as low return/low risk funds to form the bottom layer of the pyramid. The next level of the pyramid is composed of growth and income funds. The top of the pyramid contains growth funds, which are considered the highest risk/highest return class of Putnam's mutual funds. Allocations do not need to follow the typical pyramid structure, where one would assume the percent allocated to each asset class decreases as risk increases (Fisher, 1997:15-16). Instead, allocations are normally based on the investor and investment adviser's personal decisions. For example, 40% of the assets might go to growth, 20% to growth and income, 20% to tax-free funds, and 20% to income funds. Specific funds are chosen after the investor chooses investment classes; however, the selection of specific mutual funds ignores covariance between layers of the pyramid. Therefore, some model portfolios could be sub-optimal compared to others, while all of them may be sub-optimal when compared to the mean variance efficient frontier (Fisher, 1997:15).

## 2.7 Risk and Utility

### 2.7.1 Reducing Risk Over Time

Time horizon can often play a significant role in portfolio allocation. Bierman (1997:52-53) compared two theoretical investments to illustrate this point. Suppose a risk-free investment guarantees a 4% annual return. Another investment has an expected return of 12%, with an 80% probability of earning 15% and a 20% probability of a zero return. Therefore, there is a 20% chance the risky investment will earn less than the risk-free one in any given year, assuming each year is an independent event. However, over a 2-year period, there is only a 4% chance the risky investment will earn less than the risk-free one. In other words, the risky investment beats the risk-free investment 96% of the time. Over a 3-year period, the advantage of the risky investment is further magnified as it beats the risk-free investment 99.2% of the time. As long as the risky investment has a significantly higher expected return and a high probability of such a return, the risky investment will look increasingly more appealing as the time horizon increases (Bierman, 1997:52-53). Such logic is commonly found in many statistics and operations research textbooks, and it has led to the creation of investor utility functions.

### 2.7.2 Investor Utility and Portfolio Allocation

Utility functions assign a score to portfolios based on their expected return and perceived risk. The literature is replete with examples of utility functions; therefore, only a few will be discussed. Taggart (1996:246) uses a logarithmic utility function based on wealth. He assumes that utility increases less rapidly as total wealth increases:

$$U(W) = \ln(W) \tag{2.8}$$

where  $U$  represents utility, and  $W$  represents wealth. His logic is based on the theory that "the investor is truly interested in total wealth at the end of the investment horizon" (Taggart, 1996:246).

Bierman (1998:83-86) believes the expected utility of a portfolio must increase over time if risky investments are to be more desirable for long-term investors. He uses a common operations research tool known as the piecewise linear utility function to incorporate an investor's aversion to loss:

$$\begin{aligned} U(x) &= x \quad \text{if } x \geq 0 \\ U(x) &= q*x \quad \text{if } x < 0 \end{aligned} \tag{2.9}$$

where  $x$  represents change in wealth, and  $q$  represents loss aversion. This equation uses outcomes at the end of an investment time horizon to calculate overall utility of risky investments. The probability that the risky investment beats the risk-free investment increases as the time horizon increases; thus, when considering risky investments with high expected value, a long time horizon will bring greater utility to the risky investment than a short horizon (Bierman, 1998:83-86).

Another popular utility formula is (Bodie, Kane and Marcus, 2002:157):

$$U = E(R) - 0.005 * A * \sigma^2 \tag{2.10}$$

where  $U$  represents utility, 0.005 is a scaling factor,  $A$  represents the investor's risk aversion parameter, and  $\sigma$  represents the portfolio standard deviation. Changes in the scaling factor lead to many variations of this basic formula. Regardless of the variation though, the utility can change greatly depending on the time horizon used to calculate expected return and standard

deviation. However, the formula does not directly take this fact into consideration. If 1-year returns are used to calculate expected return and standard deviation, the utility for an investor with a 20-year time horizon will equal the utility for an investor with a 2-year horizon, assuming their level of risk aversion is the same.

## **2.8 Direct Selection of an Investment Alternative**

### **2.8.1 Simulation**

The portfolios along Markowitz's efficient frontier are assumed to be normal distributions, with defined parameter values for mean and standard deviation. Estimated values for future cash flows and parameters for a series of one-period returns can be combined to estimate the probability distributions of future values of investment streams (Jennings, 2003: n. pag.). The resulting probability distributions can be examined to determine the best investment strategy for individual investors. Simulation of cash flows is common practice, and there are several software packages available. Some packages, such as @Risk and Crystal Ball, allow users to build their own simulation programs. Other packages, such as Financial Engines, SCENARIOS™ - Portfolio Management, and PerTrac 2000, are specifically designed to assist in financial decisions.

Sharpe, Goldstein, and Blythe (2000) have proposed a new tool for investors known as the Distribution Builder. Studies have found that individuals vary their decisions regarding investment alternatives based on the manner in which the data is presented. Therefore, the Distribution Builder asks investors to choose from probability distributions of end-of-period wealth for a given investment cash flow. The purpose is to infer an investor's marginal utility of wealth and/or find an investment strategy that meets an individual's return distribution

requirements. Sharpe et al (2000) believe that true investor preferences are better elicited if the alternative outcomes resemble as closely as possible those in actual capital markets. They argue that their model is a method that assists in eliciting those true preferences.

Similar to the Distribution Builder, the simulation of investor cash flows into TSP accounts can provide probability distributions. Since the simulations will be among different allocations of the same investment stream, the cash flows used to develop the probability distributions will be the same. An advantage of a TSP simulation will be that investors are able to choose from return distributions of actual portfolios, so there is no need to infer preferences. The best alternative is chosen once the most preferred distribution is selected. Furthermore, a TSP simulation will provide alternative outcomes that are reasonably close to actual outcomes.

### **2.8.2 Individual Preferences Among Simulated Distributions**

Although simulations are invaluable in presenting information, many individuals may have difficulty interpreting the distributions and not feel comfortable in making a selection. Therefore, a selection model may be very beneficial. Some selection models analyze risk in terms of future values and consider risk to be unacceptable if it results in “a position with an unacceptable future net worth” (Artzner and others, 1999:205). Researchers often consider this risk-of-loss type of analysis useful for selecting from portfolios along the efficient frontier (Fabozzi, Gupta, and Markowitz, 2002:15). Individuals, however, may have more than one objective for their investments. While they may have goals they wish to achieve, they may also have minimum thresholds for their investments. The method for achieving balance between these objectives will be discussed in Chapter 3.

## **Chapter 3: Methodology**

### **3.1 Introduction**

This chapter will apply three of the mathematical models discussed in Chapter 2: optimization using Markowitz's modern portfolio theory, simulation of investor cash flows, and selection using linear programming. It begins with data collection and verification in order to verify model assumptions. Once the data is found to meet these assumptions, the creation of the optimization, simulation, and selection models for TSP investors will be discussed.

### **3.2 Optimization Model**

#### **3.2.1 Data Collection and Verification**

The historical returns for the funds in the Thrift Savings Plan (TSP) were collected primarily from the TSP website. The original three TSP funds (G, F, and C) were created in January 1988; however, the returns for January were removed from the data set because they did not represent the complete month. The S and I funds were introduced later and data was available only after May 2001. Return data for the S and I funds from February 1998 to May 2001 were approximated using the indices the funds track. The S Fund tracks the Wilshire 4500 stock index, and the I Fund tracks the Morgan Stanley Capital International Europe, Australasia, and Far East (EAFE) stock index. The data for the indices were obtained from the Wilshire (Wilshire Associates Incorporated, 2001) and Morgan Stanley (Morgan Stanley Capital International Inc., 2003) websites, respectively.

The historical returns were collected in three ways: annual returns, monthly returns, and annual mean monthly returns. The annual and annual mean monthly returns were represented by only 15 data points for each fund, whereas the monthly returns contained 191 points. Since the

size of the data set is important when considering covariance, the monthly return data were used for the research. Besides the covariance issue, the normal quantile plots show the monthly data were close to their expected values and the histograms were mound-shaped. The fund data and plots are located in Appendix A.

Although the Markowitz model is based on the assumption that the return data are drawn from a normal distribution, a true normal distribution of the data is technically infeasible since the lowest possible return on any investment is limited to  $-100\%$ . Therefore, further analysis was conducted to determine whether a normal approximation of the return data was appropriate. The Shapiro-Wilk Test for Normality was used to accomplish this task (Conover, 1980:363). With this test, the null and alternative hypotheses are stated as follows:

$H_0$ : The population represented by the monthly return data is normal.

$H_A$ : The population represented by the monthly return data is not normal.

Therefore, the alternative hypothesis cannot be assumed to be correct unless there is convincing evidence of its truth. To test for normality, the Shapiro-Wilk Test uses a test statistic known as  $W$ , which is defined as:

$$W = \frac{(\sum_{i=1}^n a_i x_i)^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (3.1)$$

where  $x_i$  are the ordered sample values ( $x_1$  is the smallest);  $a_i$  are constants generated from the mean, variances, and covariances of the order statistics of a sample of size  $n$  from a normal distribution; and  $\bar{x}$  is the sample mean (National Institute of Standards and Technology:2003, n.

pag.). The corresponding p-value (observed level of significance) was calculated for the data set for each fund and the defined level of significance,  $\alpha$ , was set at 0.05. Common values for  $\alpha$  include 0.1, 0.05, and 0.01. This value was user defined, and 0.05 was selected since normality tests tend to reject the normality assumption easily. There are cases of the Shapiro-Wilk Test where p-values of 0.01 and even 0.001 were selected, and the normality assumption was rejected. If the p-value for the test statistic is greater than 0.05, then confidence in the alternative is less than 95%. The null hypothesis will not be rejected unless there is at least 95% confidence in the alternative. As shown in Table 3.1, the results of the tests showed that the null hypothesis could not be rejected for any of the funds; therefore, the assumption of normality appears to be valid.

**Table 3.1 Shapiro-Wilk Test Results**

	<b>W</b>	<b>p-value</b>
<b>G Fund</b>	0.9729	0.0578
<b>F Fund</b>	0.9888	0.9000
<b>C Fund</b>	0.9832	0.5612
<b>S Fund</b>	0.9760	0.1403
<b>I Fund</b>	0.9899	0.9383

It is interesting to note that the lowest risk fund (G Fund) was the one in which the p-value came closest to rejecting the assumption of normality and that the highest risk fund (I Fund) was at the other end of the spectrum. The G Fund is the only fund where returns are established by the government and guaranteed to be nonnegative. The fact that the null hypothesis of normality cannot be rejected is an accomplishment in itself. Statistical tests of normality are sensitive, meaning “they tend to reject the hypothesis of normality for any distribution that is not perfectly symmetrical and mound-shaped” (McClave and others,

2001:236). Results of the Shapiro-Wilk Test for Normality for all the funds are located in Appendix A along with the histograms and quantile plots of return data.

### 3.2.2 Model Design

An Excel spreadsheet was used to calculate the mean monthly return and sample standard deviation for each of the funds in the TSP, as well as the covariance between fund pairs. These variables, along with fund weights, were used to calculate portfolio risk and return according to the formulas stated in Chapter 2 and as follows:

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (2.1)$$

$$\sigma_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}(r_i, r_j)} \quad (2.4)$$

The variables used in these equations were previously defined in Chapter 2.

Using Excel's solver add-in, an optimized portfolio was created that minimized risk (standard deviation) for a specific level of return. This was accomplished by solving the above equations with various fund weights and finding the best set of weights for each level of return. Recall that the fund weights represent the investment percentage allocated to a specific fund. Individual fund weights were set at any integer value between 0 and 100%. The integer constraint was added since TSP fund allocations must be made in whole numbers. Additionally, the sum of the individual fund weights for each portfolio was set to 1.

The range of possible monthly returns varied from 0.5469% (G Fund) to 1.058% (S Fund). An infinite number of additional portfolios could have been created between these

extremes; however, additional portfolios were spaced at intervals of approximately 0.05%.

Therefore, the alternative set consisted of 13 different portfolios as shown in Table 3.2. Most portfolios were not spaced by 0.05% exactly due to the integer constraint on the percent allocated to each fund. These portfolio alternatives were then used for the simulation and selection portions of this research. The Excel answer reports for each portfolio solution are located in Appendix B.

**Table 3.2 Portfolio Alternatives**

	<b>Return</b>	<b><math>\sigma</math></b>	<b>G Fund</b>	<b>F Fund</b>	<b>C Fund</b>	<b>S Fund</b>	<b>I Fund</b>
<b>Portfolio 1</b>	0.5469%	0.1145%	100%	0%	0%	0%	0%
<b>Portfolio 2</b>	0.5499%	0.1221%	97%	3%	0%	0%	0%
<b>Portfolio 3</b>	0.6006%	0.4091%	75%	18%	6%	1%	0%
<b>Portfolio 4</b>	0.6502%	0.7580%	54%	32%	12%	2%	0%
<b>Portfolio 5</b>	0.6999%	1.1174%	29%	51%	14%	6%	0%
<b>Portfolio 6</b>	0.7503%	1.4739%	7%	66%	24%	3%	0%
<b>Portfolio 7</b>	0.7999%	1.8510%	2%	60%	34%	4%	0%
<b>Portfolio 8</b>	0.8505%	2.2783%	0%	50%	46%	4%	0%
<b>Portfolio 9</b>	0.9000%	2.7480%	3%	34%	61%	2%	0%
<b>Portfolio 10</b>	0.9507%	3.2163%	1%	24%	73%	2%	0%
<b>Portfolio 11</b>	1.0002%	3.6894%	0%	13%	85%	2%	0%
<b>Portfolio 12</b>	1.0528%	4.2059%	0%	0%	99%	1%	0%
<b>Portfolio 13</b>	1.0580%	5.2330%	0%	0%	1%	99%	0%

### 3.3 Simulation Design

After the portfolios were created, the decision maker's inputs were incorporated and a simulation model was developed to estimate probability distributions of the dollar values for individual investment streams. The decision maker's inputs included: existing TSP account value, length of remaining government career, time until retirement money will be utilized, monthly investor contributions (in dollars), agency contributions, bonus contributions, and

expected annual contribution increases. Although the model does not place limits on annual contribution totals, the limits specified by the Internal Revenue Service and referenced in Chapter 2 must be considered. As discussed in Chapter 2, this limit increases annually by \$1,000 until 2006, when the limit is \$15,000. Although future limits are indexed to inflation, the exact contribution limit after 2006 is unknown. Therefore, total annual contribution limits should be compared to expected IRS limits before any simulation is run. Unrealistic totals will require changes to some of the input variables. Additionally, the simulation model is based on the assumption that individuals will estimate their contribution amounts before selecting a portfolio. It does not attempt to make those contributions a dependent variable. Therefore, individuals looking for assistance in determining how much to contribute to the TSP cannot do so directly with this decision tool.

During the simulation, monthly returns are estimated using a normally distributed random number generator. The random number for each alternative is based on the mean percentage return and standard deviation calculated in the optimization model. The dollar return for each month is calculated based on the random return of the portfolio alternative and the amount already invested in the TSP account (previous month's total dollars plus contribution amount). Thus the dollar value for the last month of the investment horizon represents the final TSP account value. Annual increases are added to the initial contribution amount from both the individual and agency contributions. The simulation is limited to a 50-year time horizon. The IRS requires that individuals begin withdrawal from their TSP account by age 70 ½ if they have not done so already (Federal Retirement Thrift Investment Board, 2001:36), so this will allow a 20-year-old employee to estimate their TSP dollars at a retirement age of 70.

### 3.4 Selection Design

The selection model uses the simulation results to assist investors in making their choices between the investment alternatives. The model attempts to minimize risk from the perspective that unacceptable risk is “a position with an unacceptable future net worth” (Artzner and others, 1999:205). However, unacceptable future net worth can take on multiple meanings to individuals. When investors compare upside potential versus downside protection, they are weighing the advantages and disadvantages of different investments and their ability to meet two objectives.

The selection model performs a similar analysis. Investors may have a minimum goal they feel their TSP accounts should achieve. This will be called the downside return. Likewise, they may have more optimistic goals they would like to see their TSP accounts achieve. This will be called upside return. The model uses these values to select the best investment for each investor. Since each alternative will have a probability associated with achieving these goals, the best investment will be defined through the following objective function:

Maximize:

$$\sum_{i=1}^n S_i (w_D P_{iD} + w_U P_{iU}) \quad (3.2)$$

Subject to:

$$\sum_{i=1}^n S_i = 1 \quad (3.3)$$

where  $i$  is the portfolio alternative;  $n$  is the total number of alternatives;  $S_i$  is a binary variable used to select an alternative  $i$ ;  $w_D$  and  $w_U$  are the individual weights for the downside and upside returns, respectively; and  $P_{iD}$  and  $P_{iU}$  are the probabilities of investment  $i$  achieving the

downside and upside returns, respectively. The binary variable  $S_i$  is used to ensure only one portfolio alternative is selected. This function assumes investors have a linear increase in objective function value as the probability of achieving each goal increases. The best investment is the alternative with the largest weighted sum of probability of achievement for each objective. Although the ideal investment will achieve both objectives with 100% probability, individual weighting will cause the alternatives to achieve any number of objective function values.

Since the individual chooses the weights used in the objective function, the model is adaptable if an investor has only one goal. In this case, the individual would put zero weight on the second goal. Sensitivity analysis would be required if multiple investments achieve a goal with the same probability. If this occurs, the investor could choose the alternative with the highest expected dollar value. This is based on the assumption that an investor would choose the alternative with the highest expected return, given that multiple alternatives achieve the investor's goals with the same probability.

The weights and goals may be selected before or after the simulation, and they can also be adjusted to suit the investor's needs. Additionally, a probability table for various final cash flow values is available to help investors choose their weights and goals. The worksheet used for the selection model is presented in Chapter 4, along with the results of the simulation and the selection for a number of sample investors.

## **Chapter 4: Results**

### **4.1 Introduction**

All full-time employees of the federal government are eligible to participate in the Thrift Savings Program (TSP). Regardless of their employment status, military or civil service, the tool developed in this research is designed to assist every employee in making TSP investment decisions. This chapter presents four examples to demonstrate the tool's effectiveness.

1. An active duty captain, retiring from service in 14 years.
2. An active duty lieutenant colonel, retiring from service in 9 months.
3. A GS-7 civil service member, retiring from service in 20 years.
4. A GS-12 civil service member, retiring from service in 8 years.

These examples are not randomly drawn from the population of government employees. Instead, they were selected to demonstrate a broad range of application.

### **4.2 Examples**

#### **4.2.1 Active Duty Captain**

For the first example, a service member early in his career was selected to provide an example of a military individual with a long investment time horizon and potentially high portfolio value. The simulation of returns consisted of 5,000 iterations, and the input values necessary for the simulation model were directly solicited from the individual and are shown in Table 4.1.

**Table 4.1 Active Duty Captain Simulation Inputs**

Existing TSP Balance	\$14,989.00
Employee Contribution	\$379.80
Agency Contribution	\$0
Total Contribution	\$379.80
Initial Month to Begin Model (1-12)	1
Annual Contribution Increase	3.0%
Length of Remaining Government Career (Years)	14
Length of Remaining Government Career (Months)	168
Time Until Retirement (Years)	32
Time until Retirement (Months)	384

	Month	Amount
Bonus 1	3	\$5,000
Bonus 2	15	\$5,000
Bonus 3	27	\$5,000

The investor was unsure about his upside return potential, but he knew he wanted no less than \$500,000 in his TSP account when he retired. He used the probability table of potential portfolio returns located in Appendix C to choose an upside return goal of \$1.5 million. These probabilities were estimated using simulation results. The total number of times each portfolio exceeded the defined return was divided by the total number of returns for each portfolio simulation. Appendix C also contains the histograms of the 13 portfolios' potential returns. Table 4.2 reports the simulation results for the defined goals of \$500,000 and \$1.5 million.

**Table 4.2 Active Duty Captain Simulation Results**

<b>Portfolio</b>	<b>Probability of Achieving Downside Goal</b>	<b>Probability of Achieving Upside Goal</b>
<b>1</b>	1	0
<b>2</b>	1	0
<b>3</b>	1	0
<b>4</b>	1	0
<b>5</b>	0.9996	0.0314
<b>6</b>	0.9998	0.2048
<b>7</b>	0.9992	0.4286
<b>8</b>	0.9980	0.5708
<b>9</b>	0.9962	0.6664
<b>10</b>	0.9932	0.7216
<b>11</b>	0.9908	0.7594
<b>12</b>	0.9866	0.7722
<b>13</b>	0.9558	0.6862

The results of the simulation show some portfolios are dominant over others, regardless of the weight between objectives. Given these two goals, an investor seeking to maximize return would never choose Portfolios 1 through 3 since Portfolio 4 provides the same probability of achieving the downside return with a higher upside potential. Portfolio 6 is dominant over Portfolio 5, since it has a higher probability of achieving both returns; and Portfolio 12 is dominant over Portfolio 13 for the same reason. The decision for the captain was between Portfolios 4 and 6 through 12. The captain was more concerned about achieving the upside return goal, and he made this goal twice as important as the downside return. The selection model was implemented to determine the best portfolio given these weights. As shown in Table 4.3, the best portfolio for the captain was Portfolio 12 (99% C Fund, and 1% S Fund), with a 98.66% probability of achieving the downside return and a 77.22% probability of achieving the

upside return. This table was constructed by summing the weighted probabilities for each portfolio. The resulting rank ordering of the portfolios is shown in Table 4.4.

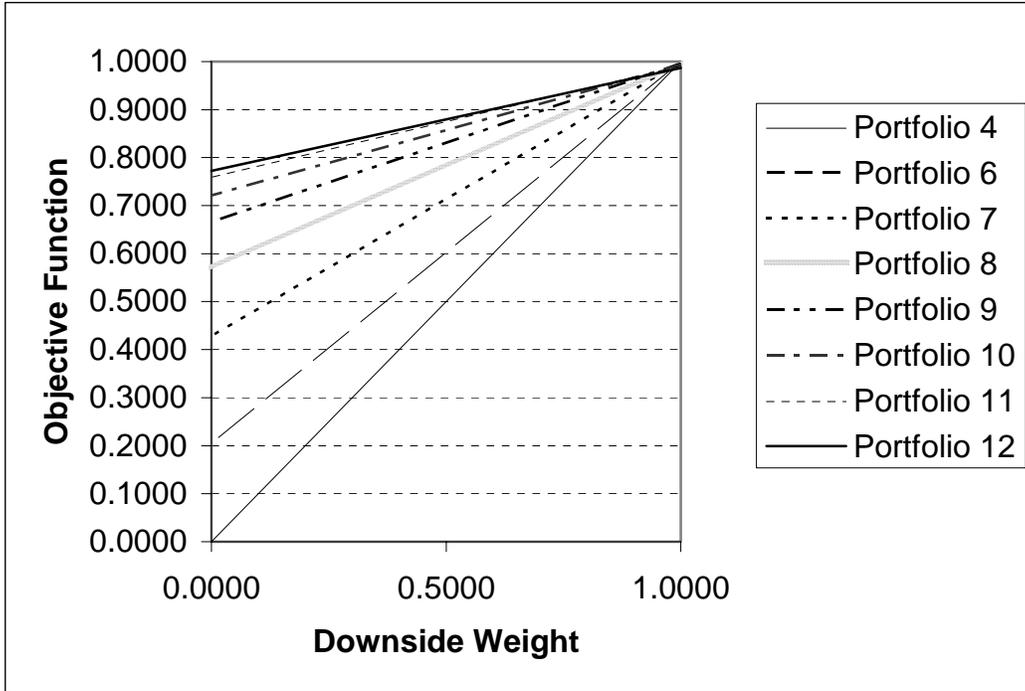
**Table 4.3 Active Duty Captain Selection Results**

<b>Portfolio</b>	<b>Probability of Achieving Downside Goal</b>	<b>Probability of Achieving Upside Goal</b>	<b>Total Score</b>
<b>1</b>	1	0	<b>1</b>
<b>2</b>	1	0	<b>1</b>
<b>3</b>	1	0	<b>1</b>
<b>4</b>	1	0	<b>1</b>
<b>5</b>	0.9996	0.0314	<b>1.0624</b>
<b>6</b>	0.9998	0.2048	<b>1.4094</b>
<b>7</b>	0.9992	0.4286	<b>1.8564</b>
<b>8</b>	0.9980	0.5708	<b>2.1396</b>
<b>9</b>	0.9962	0.6664	<b>2.3290</b>
<b>10</b>	0.9932	0.7216	<b>2.4364</b>
<b>11</b>	0.9908	0.7594	<b>2.5096</b>
<b>12</b>	0.9866	0.7722	<b>2.5310</b>
<b>13</b>	0.9558	0.6862	<b>2.3282</b>

**Table 4.4 Active Duty Captain Alternative Ranking**

<b>Rank</b>	<b>Portfolio</b>	<b>Score</b>
<b>1</b>	12	2.5310
<b>2</b>	11	2.5096
<b>3</b>	10	2.4364
<b>4</b>	9	2.3290
<b>5</b>	13	2.3282
<b>6</b>	8	2.1396
<b>7</b>	7	1.8564
<b>8</b>	6	1.4094
<b>9</b>	5	1.0624
<b>10</b>	4	1
<b>11</b>	3	1
<b>12</b>	2	1
<b>13</b>	1	1

If the investor is unsure of the weights to assign to the upside and downside goals, sensitivity analysis can be conducted to determine which alternative is best for various weighting schemes. This was done by normalizing the objective weights relative to one and plotting the new objective function versus the downside weight. The objective function only needs to be graphed as a function of this weight since the upside weight is now one minus the downside weight. The resulting sensitivity graph is shown in Figure 4.1. The graph includes only those portfolios that were not dominated.



**Figure 4.1 Active Duty Captain Sensitivity Graph**

Figure 4.1 shows Portfolio 12 is the best choice for most weighting schemes, and the alternatives become clustered as the downside weight approaches one. The graph further shows that each of these portfolios has a different slope. These varying slopes could mean one or more additional portfolios are dominated by other alternatives. This was not the case for the captain, but a later example demonstrates the occurrence of this event.

Pair-wise analysis was conducted to calculate the weights resulting in a recommendation that the investor switch portfolio choices. The steps in this analysis were as follows:

1. Use the graph to determine which portfolio intersects with the top scoring alternative.
2. Set the objective functions of the two alternatives equal to each other, leaving the weights as unknown variables:

$$w_D * P_{1D} + w_U * P_{1U} = w_D * P_{2D} + w_U * P_{2U} \quad (4.1)$$

where  $P_{1D}$  and  $P_{1U}$  are the probabilities of the first alternative achieving the downside and upside returns, respectively; and  $P_{2D}$  and  $P_{2U}$  are the probabilities of the second alternative achieving the downside and upside returns, respectively.

3. Solve for the relative weight of the return objective (downside or upside) for which the top alternative scored lower than the second alternative. This gives the relative weight of the measure when the recommendation would change and is termed the point of indifference.
4. Repeat steps 1 through 3 until all intersections have been considered.

Using this process, the point of indifference between Portfolio 12 and Portfolio 11 occurs when the downside return weight is 3.0476 times the upside return weight. Thus, the investor should not change his portfolio choice unless his downside goal weight is more than 3.0476 times the upside goal weight. The remaining pairwise comparison results are shown in Table 4.5.

**Table 4.5 Active Duty Captain Points of Indifference Between Portfolios**

<b>Portfolios</b>	<b>Downside Weight (Upside Weight = 1)</b>
<b>11 and 10</b>	<b>15.7500</b>
<b>10 and 9</b>	<b>18.4000</b>
<b>9 and 8</b>	<b>53.1111</b>
<b>8 and 7</b>	<b>118.5000</b>
<b>7 and 6</b>	<b>373.0000</b>
<b>6 and 4</b>	<b>1024.0000</b>

This type of analysis not only assists investors who are unsure about the weights between their objectives, but also provides insight to all investors. The table in Appendix C provides the probability of the 13 portfolios achieving many different objectives, thus providing further insight. The increasing portfolio variances are visible through the increasing spread of probabilities in the table. However, every portfolio has 100% probability of achieving at least

\$100,000. The most optimistic alternative is Portfolio 11, with a 50.98% probability of reaching \$2.6 million. The investor can also see the best portfolio (highest return) that guarantees a specific minimum return. For example, Portfolio 4 has the highest upside return potential with 100% probability of achieving at least \$550,000. Finally, the investor can use the table to see which goals are realistic and which are not.

For this example, Portfolio 12 is the best choice (99% C Fund, and 1% S Fund). Depending on the investor's risk aversion, Portfolios 4 or 6 through 11 could be the best alternative. If the probabilities of two alternatives achieving each objective were equal, the captain wanted the portfolio with the higher expected return. Therefore, Portfolios 1 through 3 should not be considered since they do not have enough upside potential. Portfolio 5 is dominated by Portfolio 6 since Portfolio 6 has a higher probability of achieving both objectives. Likewise Portfolio 13 should not be considered since it adds risk without improving the probability of achieving either goal. Goals may change, and the probability table provides access to the probabilities of achieving other investment goals. However, the selection model must be implemented again if the investor wishes to change goals.

#### **4.2.2 Active Duty Lieutenant Colonel**

For the second example, a service member nearing retirement from active duty was selected to provide an example of a military individual with a relatively short investment time horizon and low portfolio value. Input values necessary for the simulation model were directly solicited from the individual and are shown in Table 4.6.

**Table 4.6 Active Duty Lieutenant Colonel Simulation Inputs**

Existing TSP Balance	\$3,526.00
Employee Contribution	\$285.00
Agency Contribution	\$0
Total Contribution	\$285.00
Initial Month to Begin Model (1-12)	3
Annual Contribution Increase	0.0%
Length of Remaining Government Career (Years)	0.75
Length of Remaining Government Career (Months)	9
Time Until Retirement (Years)	17
Time until Retirement (Months)	204

This investor knew his goals for upside and downside return before the simulation was run. He wanted \$10,000 on the downside and \$20,000 on the upside. The results are shown in Table 4.7, and Appendix C contains the histograms of the 13 portfolios' potential returns.

**Table 4.7 Active Duty Lieutenant Colonel Simulation Results**

<b>Portfolio</b>	<b>Probability of Achieving Downside Goal</b>	<b>Probability of Achieving Upside Goal</b>
1	1	0
2	1	0
3	1	0.6442
4	1	0.8622
5	1	0.9076
6	1	0.9272
7	1	0.9290
8	0.9998	0.9210
9	0.9990	0.9196
10	0.9990	0.9150
11	0.9964	0.9114
12	0.9924	0.9026
13	0.9720	0.8248

According to the simulation results, Portfolios 1 through 7 have 100% probability of achieving the downside goal. The results also show that Portfolio 7 is dominant over all others, regardless of the weights that might be chosen for the objective function. Given the individual's goals, a logical investor would never choose Portfolios 1 through 6 since Portfolio 7 provides the same probability of achieving the downside return with a higher probability of achieving the upside return. Portfolio 7 is dominant over Portfolios 8 through 13 since it has a higher probability of achieving both the downside and upside returns. Therefore, this individual does not need to proceed to the selection model since Portfolio 7 (2% G Fund, 60% F Fund, 34% C Fund, and 4% S Fund) is the dominant alternative. If the investor's goals were to change, Appendix C could be utilized implement a formal selection model similar to the previous example.

#### **4.2.3 GS-7 Civil Servant**

For the third example, a civil servant early in her career was selected to provide an example of a civilian with a long investment time horizon and potentially high portfolio value. Since civilian employees are on a two-week pay period, two new assumptions are necessary to use the model. First, the model assumes the money is deposited once per month. Therefore, it is assumed that civilians will contribute to their TSP account once a month. Consequently, the model misses the compounding of the deposit for the first two-week periods and underestimates the future value of the account. Second, there are two extra paychecks per year (on average). These paychecks were split evenly among the other 12 monthly deposits represented in the model. Given these new assumptions, the input values necessary for the simulation model were solicited and adjusted as shown in Table 4.8.

**Table 4.8 GS-7 Simulation Inputs**

Existing TSP Balance	\$4,403.00
Employee Contribution	\$163.28
Agency Contribution	\$163
Total Contribution	\$326.56
Initial Month to Begin Model (1-12)	1
Annual Contribution Increase	3.0%
Length of Remaining Government Career (Years)	20
Length of Remaining Government Career (Months)	240
Time Until Retirement (Years)	35
Time until Retirement (Months)	420

As was the case with the captain, this investor was unsure about her upside return potential. She used the probability table of potential portfolio returns located in Appendix C to choose an upside return goal of \$1 million. Appendix C also contains the histograms of the 13 portfolios' potential returns. This investor had no downside return goal. She wanted the \$1 million dollars to serve as a retirement income source, and she wanted the investment with the highest probability of achieving this objective. Table 4.9 provides the ranked order of the portfolios and their probability of each alternative achieving this goal.

**Table 4.9 GS-7 Simulation Results**

<b>Rank</b>	<b>Portfolio</b>	<b>Probability of Achieving Goal</b>
<b>1</b>	12	0.8880
<b>2</b>	11	0.8850
<b>3</b>	10	0.8720
<b>4</b>	9	0.8646
<b>5</b>	8	0.8370
<b>6</b>	7	0.7812
<b>7</b>	6	0.6572
<b>8</b>	5	0.1186
<b>9</b>	4	0.0644
<b>10</b>	3	0
<b>11</b>	2	0
<b>12</b>	1	0

Since this employee had a single goal, there was no need for a formal selection process. The decision could be made directly from the ranking of alternatives. The best investment for her is Portfolio 12 (99% C Fund and 1% S Fund). However, Portfolio 11 (13% F Fund, 85% C Fund, and 2% S Fund) has almost as high of a probability of achieving her goal (88.5% versus 88.8%). The probability table and histograms in Appendix C show Portfolio 12 has higher upside return potential while Portfolio 11 reduces the risk of a low return. Either portfolio would work well towards achieving her goal, but Portfolio 12 has the highest probability of achieving it. She can easily reference the probability table to change her decision if her upside return goal changes. However, the selection model must be implemented if she wishes to add a downside return goal.

#### **4.2.4 Civil Service GS-12**

For the final example, a civil servant with 8 years of remaining service was selected to provide an example of a civilian with a medium length investment time horizon and potentially

high portfolio value. Using the same assumptions as specified for the GS-7 employee, input values are shown in Table 4.10.

**Table 4.10 GS-12 Simulation Inputs**

Existing TSP Balance	\$150,000.00
Employee Contribution	\$406.64
Agency Contribution	\$254
Total Contribution	\$660.79
Initial Month to Begin Model (1-12)	1
Annual Contribution Increase	3.0%
Length of Remaining Government Career (Years)	8
Length of Remaining Government Career (Months)	96
Time Until Retirement (Years)	15
Time until Retirement (Months)	180

The investor's upside return goal was \$1 million, while his downside goal was \$500 thousand. He also knew the downside goal was 10 times as important as the upside goal before the simulation was run. Table 4.11 reports the simulation results for these goals, and the probability table and histograms are located in Appendix C.

**Table 4.11 GS-12 Simulation Results**

<b>Portfolio</b>	<b>Probability of Achieving Downside Goal</b>	<b>Probability of Achieving Upside Goal</b>
<b>1</b>	1	0
<b>2</b>	1	0
<b>3</b>	0.9998	0
<b>4</b>	0.9970	0
<b>5</b>	0.9932	0.0050
<b>6</b>	0.9898	0.0640
<b>7</b>	0.9770	0.1814
<b>8</b>	0.9758	0.3002
<b>9</b>	0.9674	0.4200
<b>10</b>	0.9570	0.4894
<b>11</b>	0.9416	0.5342
<b>12</b>	0.9354	0.5806
<b>13</b>	0.8608	0.5150

The simulation results are slightly different than previous investors, but the interpretation remains the same. Portfolio 2 is dominant over Portfolio 1 for this investor since Portfolio 2 has a higher expected return with the same probability of achieving the downside goal. Portfolio 2 is also dominant over Portfolios 3 and 4 since it can guarantee achieving the downside while the other two alternatives cannot. However, Portfolios 1 through 4 all have zero probability of achieving \$1 million. Additionally, Portfolio 12 dominates Portfolio 13 since Portfolio 12 has a higher probability of achieving both goals. The decision for this investor is between Portfolio 2 and Portfolios 5 through 12. As shown in Table 4.12, the best portfolio for this employee was Portfolio 9 (3% G Fund, 34% F Fund, 61% C Fund, and 2% S Fund), with a 96.74% probability of achieving the downside goal and a 42% probability of achieving the upside goal.

**Table 4.12 GS-12 Selection Results**

<b>Downside Weight</b>		<b>10</b>	
<b>Upside Weight</b>		<b>1</b>	
<b>Portfolio</b>	<b>Probability of Achieving Downside Goal</b>	<b>Probability of Achieving Upside Goal</b>	<b>Total Score</b>
<b>1</b>	1	0	<b>10</b>
<b>2</b>	1	0	<b>10</b>
<b>3</b>	0.9998	0	<b>9.9980</b>
<b>4</b>	0.9970	0	<b>9.9700</b>
<b>5</b>	0.9932	0.0050	<b>9.9370</b>
<b>6</b>	0.9898	0.0640	<b>9.9620</b>
<b>7</b>	0.9770	0.1814	<b>9.9514</b>
<b>8</b>	0.9758	0.3002	<b>10.0582</b>
<b>9</b>	0.9674	0.4200	<b>10.0940</b>
<b>10</b>	0.9570	0.4894	<b>10.0594</b>
<b>11</b>	0.9416	0.5342	<b>9.9502</b>
<b>12</b>	0.9354	0.5806	<b>9.9346</b>
<b>13</b>	0.8608	0.5150	<b>9.1230</b>

This portfolio provides a balance between the ability to achieve both objectives. A more risk-averse investor would choose one of the lower portfolios, while a less risk-averse individual would consider Portfolio 10. The rank ordering of the portfolios under consideration is shown in Table 4.13.

**Table 4.13 GS-12 Alternative Ranking**

<b>Rank</b>	<b>Portfolio</b>	<b>Score</b>
<b>1</b>	9	10.0940
<b>2</b>	10	10.0594
<b>3</b>	8	10.0582
<b>4</b>	2	10
<b>5</b>	1	10
<b>6</b>	3	9.9980
<b>7</b>	4	9.9700
<b>8</b>	6	9.9620
<b>9</b>	7	9.9514
<b>10</b>	11	9.9502
<b>11</b>	5	9.9370
<b>12</b>	12	9.9346
<b>13</b>	13	9.1230

Sensitivity analysis was conducted in the same manner as with the first example in this chapter. Figure 4.2 shows the normalized objective function values as the downside goal weight changes. When the downside weight is zero, the alternatives are ranked in order of decreasing risk and return (according to Markowitz's definitions).

Some portfolios would never be considered since the slopes of each line are different. Figure 4.3 provides an enlarged view of the upper right quadrant to demonstrate this fact. Although Portfolio 11 is the second best alternative when the downside weight is zero, Portfolio 10 intersects Portfolio 12 first, specifically when the downside weight is approximately 0.8085 (4.2222 times the upside weight). Portfolio 9 intersects Portfolio 10 when the downside weight is approximately 0.8697 (6.6731 times the upside weight), and Portfolio 2 intersects Portfolio 9 when downside weight is approximately 0.9280 (12.8834 times the upside weight). Therefore, the only portfolios this employee should consider are 12, 10, 9, and 2. The other portfolios are dominated by these four, regardless of the investor's level of risk aversion.

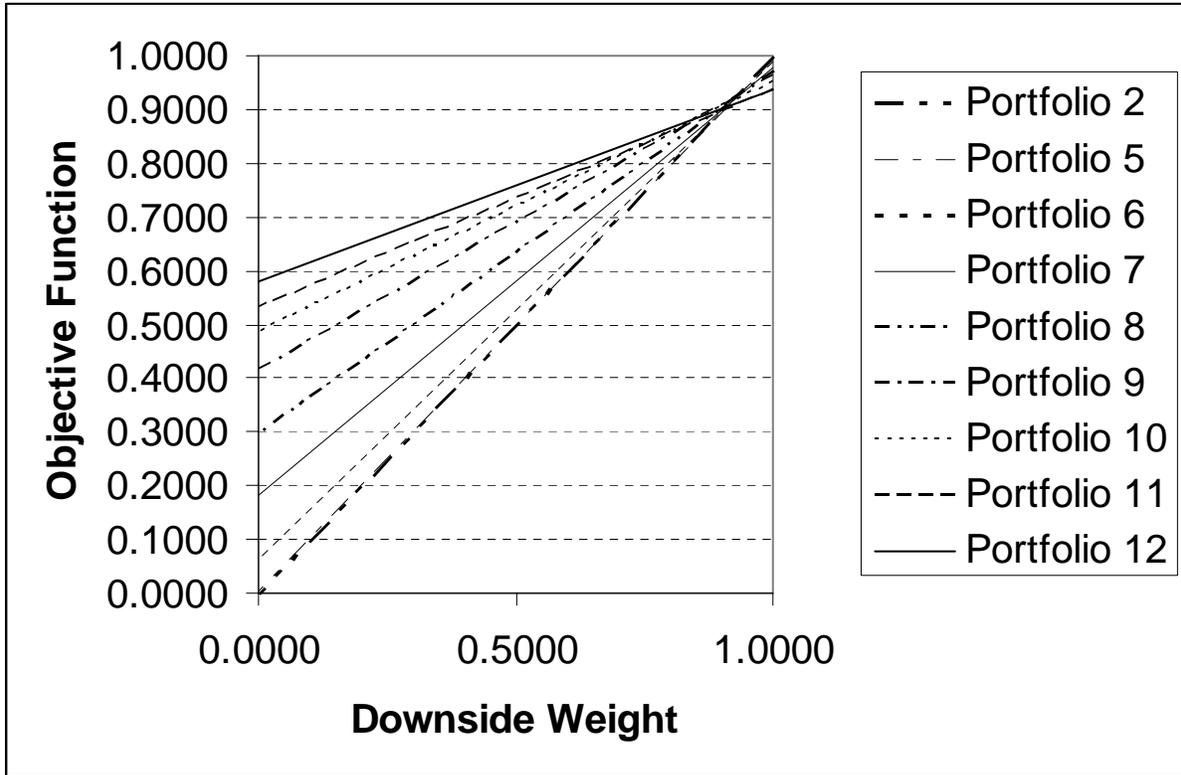


Figure 4.2 GS-12 Sensitivity Graph

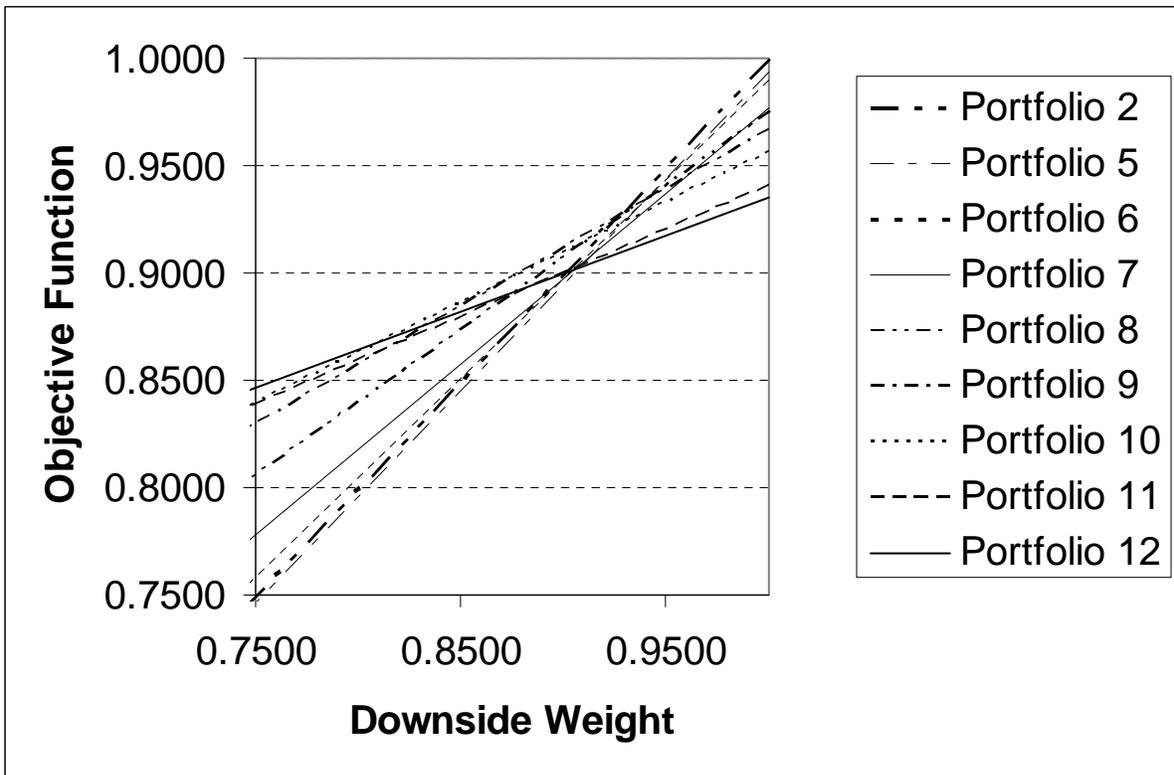


Figure 4.3 GS-12 Sensitivity Graph: Upper Right Quadrant

### 4.3 Conclusion

These cases provide results for four different investors. The tool is designed to use their inputs and calculate a recommended solution. As expected, some alternatives were unable to achieve one or both goals with acceptable probability and were dominated by other alternatives. Portfolio 13 was never considered for any of these investors, even though its mean return was often the highest. The variability of Portfolio 13 is so large that its likelihood of achieving these specific goals is too low. The unexpected result was demonstrated in the final example. Sometimes the second best alternative for one weighting scenario never became the top scoring alternative, regardless of the weight between goals. It turns out that such alternatives were dominated by pairs of other alternatives. One of these dominant alternatives had a higher probability of achieving the upside goal, and the other had a higher probability of achieving the downside goal. The examples provided in this chapter serve to demonstrate the system's adaptability. Federal government employees considering investment options should not attempt to follow these examples, since every investor has different inputs and objectives.

## **Chapter 5: Conclusions and Recommendations**

### **5.1 Research Overview**

This research involved using three distinct mathematical models to create a new tool for Thrift Savings Plan investing, the first investment tool designed specifically for the government's defined contribution retirement plan. The investment tool provides assistance to those with and without in-depth investment experience by estimating end-of-period wealth to provide insight into selecting an appropriate investment portfolio. Existing assistance provided by the Federal Thrift Investment Board does not yet fully address the ramifications of individual investment decisions.

The basis behind the optimization model is Modern Portfolio Theory, as introduced by Harry Markowitz in 1952. Many financial experts have followed the theory, while many have argued against it. The purpose of this research was not to introduce a new optimization theory, but to adopt the most appropriate existing methodology. The research highlighted why Markowitz's theory is most appropriate and demonstrated its adoption to the TSP. The second mathematical model used in this research was a simulation of investor cash flows. This model was selected for its ability to estimate probability distributions of future wealth. The research covered its construction and demonstrated its application in four distinct examples. The selection model utilized linear programming to assist investors in selecting a specific alternative. It combined the risk and return of the individual alternatives by measuring their probability of achieving specific objectives.

## 5.2 Strengths and Limitations

Decision support systems require a number of assumptions and simplifications which make the systems either stronger or weaker in specific applications. This section will discuss the strengths and limitations for this specific system. The optimization model follows Markowitz's efficient frontier. If an investor disagrees with Markowitz, then the optimization model is of no use. It is possible, however, to alter this model to follow another optimization theory, provided the necessary inputs are available. The alternatives created by the new optimization model can be used as inputs to the existing simulation model. The Markowitz model also assumes normality. Chapter 3 presented reasoning not to reject this hypothesis, but it never proved that the return data are drawn from a normal distribution. This model also assumes returns are not time dependent. An analysis of the cyclical nature of markets would likely provide a strong argument against this assumption. The expected returns and variances of the 13 portfolios are based entirely on historical data. Markowitz (Markowitz, 1952:91) believed that methods that take other information into account would be better. Arbitrage Pricing Theory has attempted to accomplish this objective with mixed results; however, the APT model demonstrates one important fact: no matter which model is chosen, investors must realize that it is no more than an estimation of actual events. No one can account for every variable, and nothing is guaranteed. This fact is most obvious with the 2003 return of the I Fund. The proposed model recommends no investment in this fund due to its low return and high volatility. However it was the second best performer in 2003, with an annual return of 37.94% (the S fund had the highest 2003 annual return at 42.92%). An investor with perfect information would have invested in the I Fund this past year, especially after the first three months of the year. The fund's performance in the final

three quarters exceeded all others. Still, many investment models and financial advisors use historical return with the caveat: past performance is no guarantee of future results.

Simulation is often useful for estimating outcomes, as demonstrated by the examples presented in Chapter 4. Although a simulation cannot provide actual probabilities, this model was able to perform 5,000 iterations for each investor in a relatively short amount of time. The primary weakness of the simulation comes from the variation of its inputs. Contribution amounts are subject to numerous factors, such as employee financial standing, promotion, annual cost of living pay raises, changes in agency contribution policies, IRS limitations, career length, and initial withdrawal age. These factors, and many others, make it impossible to know the exact amount of future contributions. Simplifications must be made, but the outcomes of the model become less accurate as actual contributions stray from those estimated.

The selection model uses two objectives, and Chapter 4 presented how it can be adopted to an investor with only one. However, investors with more than two objectives would need to alter this selection model to fit their needs. The selection model also assumes a linear increase in investor values as the probability of achieving a goal increases. This makes the model easy to implement, but in truth, very few investors would have this type of value function. For example, would every investor have the same increase in value by increasing his probability of achieving an objective from 40% to 50% as he would by increasing his probability from 90% to 100%? The shape of investor value functions are most likely not linear. Furthermore, every investor's value function is his own. Without the assumption, this model would need to recreate a value function every time another investor used the model. While this may be a weakness, the model rectifies potential pitfalls by allowing the investors to place weights on the two objectives. Trial and error can be used to adjust weights and assist an investor in selecting the most appropriate

balance between the two objectives. However, this too can be a drawback if investors manipulate weights to achieve a specific portfolio.

### **5.3 Conclusion**

While this investment model has weaknesses, it is safe to assume that no single model will satisfy everyone. In a broad context, this research has demonstrated that it is possible to build a decision support tool for TSP investors. Specifically, it has shown how the decision can be split into separate components. Perhaps the most difficult part of an investment decision is finding which portfolios are optimum for specific risk and return combinations. Using the assumption that investors will act rationally (Markowitz's fifth assumption), this part of the process can be completed before the investor ever considers this decision. The simulation and selection models require only a few user inputs, and they can be performed on any personal computer. Thus the entire decision tool can be used by individual investors without any external assistance.

This research had a few findings in addition to successfully building a tool for supporting individuals choosing their TSP allocations. First, application of Modern Portfolio Theory to the TSP has shown that one fund should not be part of any investor's present portfolio. Historical returns show the I Fund is a high risk/low return investment. This and its covariance with the other funds do not make it a viable investment to reduce portfolio risk or increase portfolio expected return. Thus the I fund is not part of any of the 13 portfolio alternatives used in this research. This finding is based on historical data, and the argument against using this information was discussed earlier in this chapter. Only time will verify the validity of this conclusion.

Second, Chapter 4 demonstrated why some portfolios should never be considered for investors with specific goals, regardless of the relative weight of the two goals. A few portfolios were dominated completely by a single portfolio alternative. This occurred when one portfolio had a higher probability of achieving both objectives, as shown in the simulation results in Chapter 4. Other portfolios were dominated by multiple portfolios, as the sensitivity analyses in Chapter 4 demonstrated. Finally, this research has demonstrated the application of risk as it relates to the probability of achieving a specific future net worth. Investments with low standard deviation, considered low risk to most people, can be considered risky investments when their probability of achieving an objective is considered. Thus, risk can be high due to the traditionally used variability of return, or risk can be high due to a low expected return, as demonstrated in this research.

#### **5.4 Recommendations for Future Research**

The potential future research concerning this topic is practically limitless. There are literally hundreds of methods for portfolio alternative generation, and further research may find one that is more applicable than the one used. Entire investor assets could be analyzed, instead of just TSP investment. The selection model used in this research was chosen for its broad applicability and ease of use. A more complex selection model could be used to provide detailed advice to individuals. In fact, an entirely new decision tool could be created if the government wished to provide professional financial advice. Further research could move beyond the decision tool itself and concentrate on its uses and benefits. For example, if the government wishes to improve employee retention, assisting in their TSP investment decisions could be a

step in the right direction. Future research may be able to find the benefit of providing such a program.

## **Appendix A. Thrift Savings Plan Fund Data and Analysis**

**Table A.1 Historical Return Data**

(Note: Items in bold represent index returns.)

Date	G	F	C	4500/S	EAFE/I
19880229	0.6200%	0.8100%	4.8200%	<b>6.5800%</b>	<b>6.6688%</b>
19880331	0.6600%	-0.8000%	-3.4700%	<b>1.8400%</b>	<b>6.1456%</b>
19880429	0.6800%	-0.4600%	0.7300%	<b>1.2700%</b>	<b>1.4509%</b>
19880531	0.7100%	-0.6300%	1.4200%	<b>-1.5300%</b>	<b>-3.2037%</b>
19880630	0.7200%	1.9700%	4.0800%	<b>6.1700%</b>	<b>-2.6355%</b>
19880729	0.7200%	-0.4900%	-0.2400%	<b>-1.5100%</b>	<b>3.1374%</b>
19880831	0.7600%	0.3300%	-2.7400%	<b>-1.8500%</b>	<b>-6.5021%</b>
19880930	0.7600%	2.0700%	4.1200%	<b>2.8200%</b>	<b>4.3681%</b>
19881031	0.7500%	1.6800%	2.5300%	<b>-0.2400%</b>	<b>8.5579%</b>
19881130	0.6800%	-1.0900%	-1.2300%	<b>-1.9400%</b>	<b>5.9566%</b>
19881230	0.7400%	0.3100%	1.7800%	<b>3.1800%</b>	<b>0.5557%</b>
19890131	0.7600%	1.2700%	7.1400%	<b>5.4800%</b>	<b>1.7611%</b>
19890228	0.6700%	-0.6800%	-2.5100%	<b>0.3500%</b>	<b>0.5143%</b>
19890331	0.7800%	0.5000%	2.2100%	<b>2.2600%</b>	<b>-1.9626%</b>
19890428	0.7500%	2.0500%	5.1400%	<b>4.3900%</b>	<b>0.9275%</b>
19890531	0.7600%	2.4200%	3.9800%	<b>3.6800%</b>	<b>-5.4421%</b>
19890630	0.7000%	3.1900%	-0.5800%	<b>-0.4800%</b>	<b>-1.6815%</b>
19890731	0.6900%	2.0600%	8.8300%	<b>5.0400%</b>	<b>12.5570%</b>
19890831	0.6600%	-1.4800%	1.9800%	<b>3.0200%</b>	<b>-4.4971%</b>
19890929	0.6800%	0.3700%	-0.2900%	<b>0.6500%</b>	<b>4.5531%</b>
19891031	0.7100%	2.4500%	-2.3300%	<b>-4.4100%</b>	<b>-4.0181%</b>
19891130	0.6500%	0.8600%	2.0500%	<b>1.1100%</b>	<b>5.0262%</b>
19891229	0.6700%	0.1600%	2.3700%	<b>0.9800%</b>	<b>3.6929%</b>
19900131	0.6800%	-1.3800%	-6.5900%	<b>-8.5300%</b>	<b>-3.7209%</b>
19900228	0.6400%	0.2100%	1.2600%	<b>2.1100%</b>	<b>-6.9795%</b>
19900330	0.7200%	0.0100%	2.6400%	<b>2.0900%</b>	<b>-10.4204%</b>
19900430	0.7100%	-0.9400%	-2.5200%	<b>-3.9500%</b>	<b>-0.7933%</b>
19900531	0.7600%	2.8000%	9.4400%	<b>7.4300%</b>	<b>11.4133%</b>
19900629	0.7100%	1.5600%	-0.7100%	<b>0.1000%</b>	<b>-0.8818%</b>
19900731	0.7200%	1.2400%	-0.3600%	<b>-2.5000%</b>	<b>1.4085%</b>
19900831	0.7200%	-1.4200%	-8.6500%	<b>-10.0500%</b>	<b>-9.7098%</b>
19900928	0.7300%	0.8100%	-4.8500%	<b>-6.7600%</b>	<b>-13.9365%</b>
19901031	0.7600%	1.3200%	-0.4600%	<b>-3.9000%</b>	<b>15.5793%</b>
19901130	0.7000%	2.1500%	6.3600%	<b>7.5700%</b>	<b>-5.9003%</b>
19901231	0.7000%	1.4600%	2.7200%	<b>3.8300%</b>	<b>1.6272%</b>
19910131	0.6900%	1.1500%	4.5500%	<b>6.0100%</b>	<b>3.2306%</b>
19910228	0.6200%	0.8600%	7.0700%	<b>9.1000%</b>	<b>10.7190%</b>
19910328	0.6800%	0.6700%	2.4000%	<b>4.3800%</b>	<b>-6.0045%</b>
19910430	0.6600%	1.0500%	0.1800%	<b>0.5700%</b>	<b>0.9842%</b>
19910531	0.6800%	0.5700%	4.3000%	<b>3.5000%</b>	<b>1.0463%</b>
19910628	0.6600%	-0.0100%	-4.4900%	<b>-4.3900%</b>	<b>-7.3475%</b>
19910731	0.6900%	1.4000%	4.6300%	<b>4.9300%</b>	<b>4.9121%</b>
19910830	0.6900%	2.1200%	2.3700%	<b>3.4800%</b>	<b>-2.0332%</b>
19910930	0.6400%	1.9900%	-1.6300%	<b>0.4300%</b>	<b>5.6374%</b>
19911031	0.6200%	1.0900%	1.3900%	<b>3.1100%</b>	<b>1.4158%</b>
19911129	0.6100%	0.8900%	-3.9600%	<b>-3.3600%</b>	<b>-4.6687%</b>
19911231	0.6200%	2.9600%	11.4100%	<b>9.9000%</b>	<b>5.1659%</b>

Date	G	F	C	4500/S	EAFE/I
19920131	0.5700%	-1.3500%	-1.8900%	<b>3.5100%</b>	<b>-2.1341%</b>
19920228	0.5600%	0.6600%	1.2900%	<b>1.4800%</b>	<b>-3.5793%</b>
19920331	0.6200%	-0.5300%	-1.9100%	<b>-3.5700%</b>	<b>-6.6022%</b>
19920430	0.6200%	0.6700%	2.9100%	<b>-2.0900%</b>	<b>0.4704%</b>
19920529	0.6400%	1.8400%	0.4900%	<b>1.0700%</b>	<b>6.6934%</b>
19920630	0.6000%	1.3600%	-1.4500%	<b>-3.0600%</b>	<b>-4.7400%</b>
19920731	0.6000%	2.0000%	4.1100%	<b>3.7000%</b>	<b>-2.5593%</b>
19920831	0.5700%	1.0000%	-2.0200%	<b>-2.2100%</b>	<b>6.2721%</b>
19920930	0.5400%	1.1500%	1.1500%	<b>1.5600%</b>	<b>-1.9747%</b>
19921030	0.5500%	-1.3000%	0.4200%	<b>2.6600%</b>	<b>-5.2455%</b>
19921130	0.5600%	0.0100%	3.3900%	<b>5.8500%</b>	<b>0.9410%</b>
19921231	0.5800%	1.5400%	1.2100%	<b>2.8800%</b>	<b>0.5172%</b>
19930129	0.5800%	1.8800%	0.8600%	<b>2.1800%</b>	<b>-0.0123%</b>
19930226	0.4900%	1.7300%	1.3500%	<b>-1.6600%</b>	<b>3.0207%</b>
19930331	0.5200%	0.4100%	2.0900%	<b>3.5300%</b>	<b>8.7166%</b>
19930430	0.5100%	0.6700%	-2.3900%	<b>-3.2700%</b>	<b>9.4903%</b>
19930528	0.5100%	0.1000%	2.6600%	<b>4.1200%</b>	<b>2.1120%</b>
19930630	0.5100%	1.7900%	0.3200%	<b>0.8200%</b>	<b>-1.5602%</b>
19930730	0.4900%	0.5500%	-0.3800%	<b>0.5800%</b>	<b>3.5005%</b>
19930831	0.4900%	1.7200%	3.7800%	<b>4.5400%</b>	<b>5.3983%</b>
19930930	0.4500%	0.2600%	-0.7600%	<b>2.0500%</b>	<b>-2.2509%</b>
19931029	0.4700%	0.3800%	2.0400%	<b>1.1800%</b>	<b>3.0818%</b>
19931130	0.4500%	-0.8400%	-0.9300%	<b>-3.3600%</b>	<b>-8.7411%</b>
19931231	0.4900%	0.5200%	1.2000%	<b>3.3800%</b>	<b>7.2207%</b>
19940131	0.5100%	1.3300%	3.4000%	<b>2.7500%</b>	<b>8.4545%</b>
19940228	0.4300%	-1.7200%	-2.7000%	<b>-1.3800%</b>	<b>-0.2770%</b>
19940331	0.5200%	-2.4500%	-4.3900%	<b>-4.8900%</b>	<b>-4.3071%</b>
19940429	0.5600%	-0.8100%	1.2800%	<b>0.6000%</b>	<b>4.2429%</b>
19940531	0.6000%	-0.0200%	1.6600%	<b>-0.5400%</b>	<b>-0.5741%</b>
19940630	0.5900%	-0.2400%	-2.4700%	<b>-3.1200%</b>	<b>1.4132%</b>
19940729	0.6200%	1.9700%	3.2700%	<b>2.7000%</b>	<b>0.9617%</b>
19940831	0.6000%	0.1300%	4.1100%	<b>4.7700%</b>	<b>2.3675%</b>
19940930	0.5900%	-1.4700%	-2.4400%	<b>-0.6800%</b>	<b>-3.1496%</b>
19941031	0.6500%	-0.1000%	2.2400%	<b>0.4300%</b>	<b>3.3300%</b>
19941130	0.6400%	-0.2300%	-3.6200%	<b>-4.0400%</b>	<b>-4.8060%</b>
19941230	0.6800%	0.6900%	1.4900%	<b>1.1700%</b>	<b>0.6262%</b>
19950131	0.6700%	1.9800%	2.5800%	<b>0.9700%</b>	<b>-3.8415%</b>
19950228	0.5900%	2.3800%	3.8700%	<b>4.2500%</b>	<b>-0.2870%</b>
19950331	0.6200%	0.6000%	2.9400%	<b>2.2000%</b>	<b>6.2371%</b>
19950428	0.6000%	1.3800%	2.9400%	<b>1.2900%</b>	<b>3.7609%</b>
19950531	0.6100%	3.8400%	3.9800%	<b>2.2300%</b>	<b>-1.1921%</b>
19950630	0.5300%	0.7100%	2.3100%	<b>4.8200%</b>	<b>-1.7537%</b>
19950731	0.5500%	-0.2300%	3.3000%	<b>5.8800%</b>	<b>6.2256%</b>
19950831	0.5600%	1.2100%	0.2600%	<b>2.3300%</b>	<b>-3.8146%</b>
19950929	0.5300%	0.9500%	4.1900%	<b>2.7100%</b>	<b>1.9530%</b>
19951031	0.5400%	1.2800%	-0.3600%	<b>-2.6800%</b>	<b>-2.6880%</b>
19951130	0.5100%	1.4900%	4.3800%	<b>4.2500%</b>	<b>2.7824%</b>
19951229	0.5000%	1.3900%	1.9200%	<b>1.2600%</b>	<b>4.0290%</b>

Date	G	F	C	4500/S	EAFE/I
19960131	0.4900%	0.6600%	3.4100%	<b>1.0300%</b>	<b>0.4105%</b>
19960229	0.4600%	-1.7500%	0.9100%	<b>3.2500%</b>	<b>0.3381%</b>
19960329	0.5400%	-0.6800%	0.9700%	<b>1.5300%</b>	<b>2.1236%</b>
19960430	0.5400%	-0.5600%	1.4700%	<b>4.6500%</b>	<b>2.9073%</b>
19960531	0.5800%	-0.1100%	2.5600%	<b>3.1500%</b>	<b>-1.8402%</b>
19960628	0.5700%	1.3400%	0.3800%	<b>-3.1400%</b>	<b>0.5628%</b>
19960731	0.5800%	0.2700%	-4.3900%	<b>-7.5100%</b>	<b>-2.9227%</b>
19960830	0.5800%	-0.1800%	2.0700%	<b>5.4000%</b>	<b>0.2191%</b>
19960930	0.5800%	1.7200%	5.6000%	<b>4.7700%</b>	<b>2.6566%</b>
19961031	0.5800%	2.2100%	2.7400%	<b>-1.4800%</b>	<b>-1.0233%</b>
19961129	0.5300%	1.6900%	7.5400%	<b>4.2200%</b>	<b>3.9788%</b>
19961231	0.5300%	-0.9300%	-1.9700%	<b>0.9000%</b>	<b>-1.2864%</b>
19970131	0.5600%	0.3000%	6.2200%	<b>3.2300%</b>	<b>-3.4998%</b>
19970228	0.5100%	0.2400%	0.7900%	<b>-2.1100%</b>	<b>1.6356%</b>
19970331	0.5700%	-1.1100%	-4.1300%	<b>-5.2200%</b>	<b>0.3621%</b>
19970430	0.5800%	1.4900%	6.0000%	<b>0.2500%</b>	<b>0.5306%</b>
19970530	0.5800%	0.9400%	6.0700%	<b>9.9500%</b>	<b>6.5074%</b>
19970630	0.5600%	1.1800%	4.4500%	<b>4.7400%</b>	<b>5.5147%</b>
19970731	0.5700%	2.6900%	7.9400%	<b>6.8800%</b>	<b>1.6178%</b>
19970829	0.5300%	-0.8600%	-5.5900%	<b>0.9100%</b>	<b>-7.4684%</b>
19970930	0.5400%	1.4800%	5.4600%	<b>6.9300%</b>	<b>5.6019%</b>
19971031	0.5400%	1.4500%	-3.3800%	<b>-3.8100%</b>	<b>-7.6865%</b>
19971128	0.5000%	0.4600%	4.6100%	<b>0.0300%</b>	<b>-1.0194%</b>
19971231	0.5200%	1.0100%	1.7100%	<b>2.4500%</b>	<b>0.8722%</b>
19980130	0.5100%	1.2800%	1.1200%	<b>-1.4300%</b>	<b>4.5734%</b>
19980227	0.4400%	-0.0700%	7.2000%	<b>7.6500%</b>	<b>6.4166%</b>
19980331	0.5000%	0.3400%	5.1100%	<b>5.1700%</b>	<b>3.0792%</b>
19980430	0.4900%	0.5200%	1.0000%	<b>1.2700%</b>	<b>0.7915%</b>
19980529	0.5100%	0.9500%	-1.7200%	<b>-4.6700%</b>	<b>-0.4854%</b>
19980630	0.4800%	0.8500%	4.0500%	<b>1.5600%</b>	<b>0.7570%</b>
19980731	0.4900%	0.2100%	-1.0900%	<b>-5.6600%</b>	<b>1.0138%</b>
19980831	0.4900%	1.6600%	-14.4700%	<b>-19.3800%</b>	<b>-12.3889%</b>
19980930	0.4400%	2.3600%	6.3300%	<b>7.2200%</b>	<b>-3.0658%</b>
19981030	0.4100%	-0.5200%	8.1900%	<b>5.6800%</b>	<b>10.4240%</b>
19981130	0.4200%	0.5600%	6.0400%	<b>6.2200%</b>	<b>5.1231%</b>
19981231	0.4300%	0.3000%	5.7600%	<b>8.4600%</b>	<b>3.9450%</b>
19990129	0.4200%	0.7100%	4.1900%	<b>1.6100%</b>	<b>-0.2953%</b>
19990226	0.3800%	-1.7400%	-3.0900%	<b>-5.4400%</b>	<b>-2.3833%</b>
19990331	0.4700%	0.5400%	3.9900%	<b>3.9000%</b>	<b>4.1742%</b>
19990430	0.4600%	0.2900%	3.8600%	<b>7.9800%</b>	<b>4.0519%</b>
19990528	0.4700%	-0.8900%	-2.3600%	<b>-0.8700%</b>	<b>-5.1499%</b>
19990630	0.4900%	-0.3300%	5.5400%	<b>4.2000%</b>	<b>3.8987%</b>
19990730	0.5200%	-0.4300%	-3.1400%	<b>-3.0300%</b>	<b>2.9723%</b>
19990831	0.5300%	-0.0500%	-0.5000%	<b>-2.4900%</b>	<b>0.3652%</b>
19990930	0.5100%	1.1500%	-2.7800%	<b>-0.7800%</b>	<b>1.0066%</b>
19991029	0.5300%	0.3800%	6.3400%	<b>5.1100%</b>	<b>3.7456%</b>
19991130	0.5100%	-0.0100%	2.0000%	<b>8.4400%</b>	<b>3.4745%</b>
19991231	0.5400%	-0.4500%	5.9000%	<b>13.7800%</b>	<b>8.9752%</b>

Date	G	F	C	4500/S	EAFE/I
20000131	0.5600%	-0.3400%	-5.0300%	<b>-1.2000%</b>	<b>-6.3539%</b>
20000229	0.5300%	1.2200%	-1.9300%	<b>15.5500%</b>	<b>2.6920%</b>
20000331	0.5500%	1.3200%	9.7400%	<b>-3.7000%</b>	<b>3.8764%</b>
20000428	0.5200%	-0.2900%	-2.9800%	<b>-12.0300%</b>	<b>-5.2623%</b>
20000531	0.5400%	-0.0300%	-2.0500%	<b>-7.3800%</b>	<b>-2.4424%</b>
20000630	0.5300%	2.0700%	2.4400%	<b>12.0100%</b>	<b>3.9108%</b>
20000731	0.5300%	0.8900%	-1.5600%	<b>-2.8400%</b>	<b>-4.1925%</b>
20000831	0.5200%	1.4600%	6.1900%	<b>11.1600%</b>	<b>0.8680%</b>
20000929	0.4900%	0.6400%	-5.2700%	<b>-4.0700%</b>	<b>-4.8692%</b>
20001031	0.5100%	0.6600%	-0.4000%	<b>-8.1700%</b>	<b>-2.3622%</b>
20001130	0.4800%	1.6500%	-7.8700%	<b>-17.0300%</b>	<b>-3.7500%</b>
20001229	0.4800%	1.8600%	0.5000%	<b>6.3300%</b>	<b>3.5544%</b>
20010131	0.4600%	1.6500%	3.5500%	<b>5.4400%</b>	<b>-0.0516%</b>
20010228	0.4200%	0.8700%	-9.1200%	<b>-12.1500%</b>	<b>-7.4968%</b>
20010330	0.4500%	0.5100%	-6.3300%	<b>-9.1800%</b>	<b>-6.6661%</b>
20010430	0.4300%	-0.4200%	7.7800%	<b>10.5800%</b>	<b>6.9492%</b>
20010531	0.4700%	0.6100%	0.6500%	1.4200%	-4.1300%
20010629	0.4700%	0.3900%	-2.4200%	0.6600%	-3.9900%
20010731	0.4800%	2.2200%	-0.9800%	-4.1300%	-1.7900%
20010831	0.4500%	1.2000%	-6.2700%	-4.3200%	-2.5800%
20010928	0.4300%	1.1500%	-8.0500%	-12.5000%	-9.9500%
20011031	0.4100%	2.1200%	1.8500%	5.0900%	2.4700%
20011130	0.3700%	-1.3700%	7.6200%	7.8400%	3.5600%
20011231	0.4200%	-0.6100%	0.8800%	5.3100%	0.5200%
20020131	0.4500%	0.7900%	-1.4700%	-2.0200%	-5.3300%
20020228	0.4000%	0.9800%	-1.9200%	-2.6400%	0.6600%
20020328	0.4400%	-1.6600%	3.7300%	6.8300%	5.8200%
20020430	0.4600%	1.8900%	-6.0600%	-1.0600%	0.2500%
20020531	0.4500%	0.8800%	-0.7500%	-2.3900%	1.2900%
20020628	0.4300%	0.9700%	-7.1000%	-6.6700%	-3.8700%
20020731	0.4300%	1.1900%	-7.7000%	-9.9300%	-9.9900%
20020830	0.4000%	1.5800%	0.6700%	0.5800%	-0.2600%
20020930	0.3700%	1.6300%	-10.8700%	-6.8400%	-10.7500%
20021031	0.3300%	-0.4400%	8.7700%	3.3800%	5.4200%
20021129	0.3400%	-0.0100%	5.8700%	6.7600%	4.4900%
20021231	0.3800%	2.0800%	-5.8500%	-4.3200%	-3.2700%
20030131	0.3500%	0.1000%	-2.6700%	-2.3500%	-4.2400%
20030228	0.3200%	1.4100%	-1.4900%	-2.5600%	-2.2900%
20030331	0.3300%	-0.0500%	0.9700%	1.5500%	-1.9000%
20030430	0.3300%	0.8300%	8.2600%	8.3100%	9.8200%
20030530	0.3400%	1.8700%	5.2600%	9.4200%	6.0700%
20030630	0.2000%	-0.3000%	1.2000%	2.2000%	2.3000%
20030731	0.3000%	-3.4100%	1.7800%	4.6000%	2.3500%
20030829	0.4000%	0.7300%	1.9400%	4.1200%	2.3900%
20030930	0.4000%	2.6800%	-1.1400%	-1.3500%	3.0800%
20031031	0.3000%	-1.0000%	5.6800%	7.6500%	6.0600%
20031130	0.3000%	0.3000%	0.9100%	3.4700%	2.2200%
20031231	0.4900%	1.0100%	5.2400%	2.0400%	7.6800%

**Table A.2 Fund Mean Return and Standard Deviation**

<b>Fund</b>	<b>G</b>	<b>F</b>	<b>C</b>	<b>4500/S</b>	<b>EAFE/I</b>
<b>Expected Return</b>	0.5469%	0.6479%	1.05277%	1.0580%	0.5185%
<b>Standard Deviation</b>	0.1145%	1.1457%	4.2061%	5.2520%	4.9388%

**Table A.3 Covariance Matrix**

	<b>G</b>	<b>F</b>	<b>C</b>	<b>4500/S</b>	<b>EAFE/I</b>
<b>Cov With G</b>	1.3111E-06	2.3782E-06	2.9648E-06	-5.7120E-07	-1.8394E-06
<b>Cov with F</b>	2.3782E-06	1.3126E-04	8.9025E-05	4.6241E-05	4.1273E-05
<b>Cov with C</b>	2.9648E-06	8.9025E-05	1.7691E-03	1.7575E-03	1.2741E-03
<b>Cov with S</b>	-5.7120E-07	4.6241E-05	1.7575E-03	2.7584E-03	1.4460E-03
<b>Cov with I</b>	-1.8394E-06	4.1273E-05	1.2741E-03	1.4460E-03	2.4392E-03

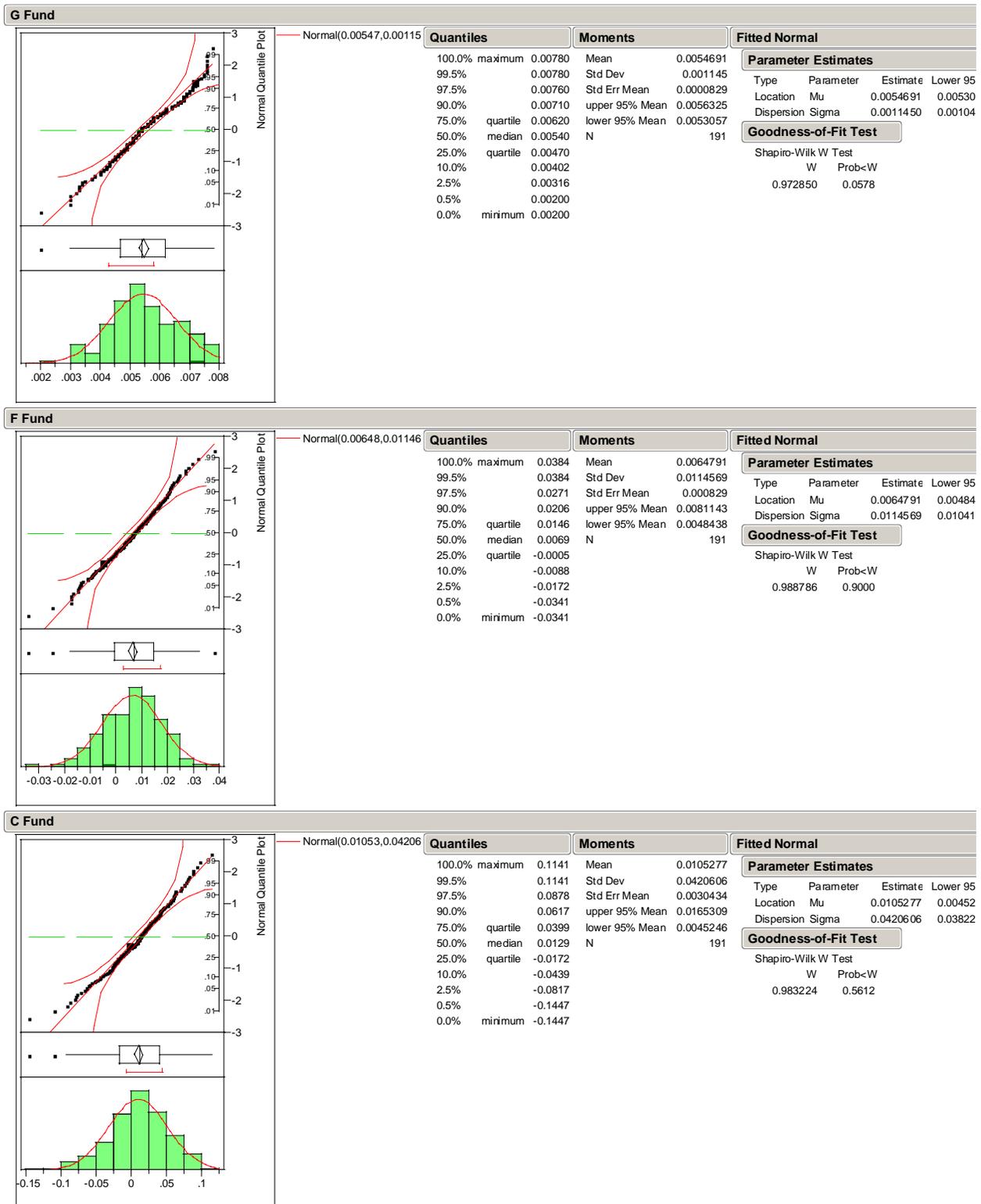
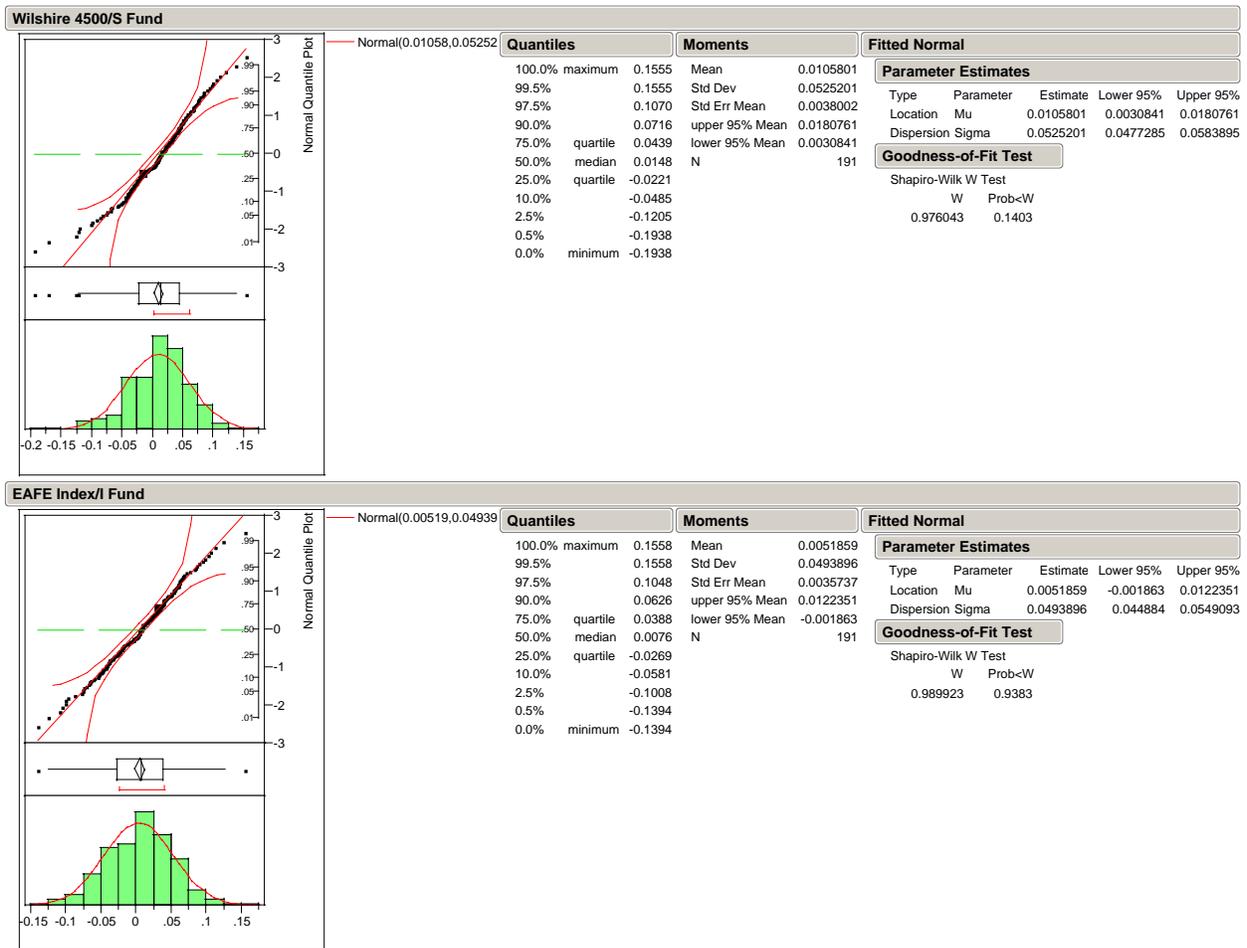


Figure A.1 G, F, and C Fund Statistics and Shapiro-Wilk Test Results



**Figure A.2 Wilshire 4500/S and EAFE Index/I Fund Statistics and Shapiro-Wilk Test Results**

## Appendix B. Excel Answer Reports

### Microsoft Excel 10.0 Answer Report

Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs

Report Created: 2/17/2004 9:41:05 PM

#### Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	0.1145%	0.1145%

#### Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F	0	0
\$D\$215	Weight C	0	0
\$D\$216	Weight S	0	0
\$D\$217	Weight I	0	0

#### Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	100.0000%	\$B\$211>=0	Not Binding	100.0000%
\$B\$212	Weight F	0.0000%	\$B\$212>=0	Binding	0.0000%
\$B\$213	Weight C	0.0000%	\$B\$213>=0	Binding	0.0000%
\$B\$214	Weight S	0.0000%	\$B\$214>=0	Binding	0.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.5469%	\$B\$216>=\$F\$213	Binding	0.0000%
\$D\$214	Weight F C Fund	0	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	0	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	0	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.1 0.5469% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:42:42 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	0.1145%	0.1221%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	0	3
\$D\$215	Weight C C Fund	0	0
\$D\$216	Weight S C Fund	0	0
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	97.0000%	\$B\$211>=0	Not Binding	97.0000%
\$B\$212	Weight F	3.0000%	\$B\$212>=0	Not Binding	3.0000%
\$B\$213	Weight C	0.0000%	\$B\$213>=0	Binding	0.0000%
\$B\$214	Weight S	0.0000%	\$B\$214>=0	Binding	0.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.5499%	\$B\$216>=\$F\$213	Binding	0.0000%
\$D\$214	Weight F C Fund	3	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	0	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	0	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.2 0.55% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:43:34 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	0.1221%	0.4091%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	3	18
\$D\$215	Weight C C Fund	0	6
\$D\$216	Weight S C Fund	0	1
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	75.0000%	\$B\$211>=0	Not Binding	75.0000%
\$B\$212	Weight F	18.0000%	\$B\$212>=0	Not Binding	18.0000%
\$B\$213	Weight C	6.0000%	\$B\$213>=0	Not Binding	6.0000%
\$B\$214	Weight S	1.0000%	\$B\$214>=0	Not Binding	1.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.6006%	\$B\$216>=\$F\$213	Not Binding	0.0006%
\$D\$214	Weight F C Fund	18	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	6	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	1	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.3 0.6% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:44:09 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	0.4091%	0.7580%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	18	32
\$D\$215	Weight C C Fund	6	12
\$D\$216	Weight S C Fund	1	2
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	54.0000%	\$B\$211>=0	Not Binding	54.0000%
\$B\$212	Weight F	32.0000%	\$B\$212>=0	Not Binding	32.0000%
\$B\$213	Weight C	12.0000%	\$B\$213>=0	Not Binding	12.0000%
\$B\$214	Weight S	2.0000%	\$B\$214>=0	Not Binding	2.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.6502%	\$B\$216>=\$F\$213	Not Binding	0.0002%
\$D\$214	Weight F C Fund	32	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	12	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	2	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.4 0.65% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:44:54 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	0.7580%	1.1174%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	32	51
\$D\$215	Weight C C Fund	12	14
\$D\$216	Weight S C Fund	2	6
\$D\$217	Weight I C Fund	0	2.2351E-12

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	29.0000%	\$B\$211>=0	Not Binding	29.0000%
\$B\$212	Weight F	51.0000%	\$B\$212>=0	Not Binding	51.0000%
\$B\$213	Weight C	14.0000%	\$B\$213>=0	Not Binding	14.0000%
\$B\$214	Weight S	6.0000%	\$B\$214>=0	Not Binding	6.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.6999%	\$B\$216>=\$F\$213	Binding	0.0000%
\$D\$214	Weight F C Fund	51	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	14	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	6	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	2.2351E-12	\$D\$217=integer	Binding	0

**Figure B.5 0.7% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:45:43 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	1.1174%	1.4739%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	51	66
\$D\$215	Weight C C Fund	14	24
\$D\$216	Weight S C Fund	6	3
\$D\$217	Weight I C Fund	2.2351E-12	2.2351E-12

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	7.0000%	\$B\$211>=0	Not Binding	7.0000%
\$B\$212	Weight F	66.0000%	\$B\$212>=0	Not Binding	66.0000%
\$B\$213	Weight C	24.0000%	\$B\$213>=0	Not Binding	24.0000%
\$B\$214	Weight S	3.0000%	\$B\$214>=0	Not Binding	3.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.7503%	\$B\$216>=\$F\$213	Not Binding	0.0003%
\$D\$214	Weight F C Fund	66	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	24	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	3	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	2.2351E-12	\$D\$217=integer	Binding	0

**Figure B.6 0.75% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:46:40 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	1.4739%	1.8510%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	66	60
\$D\$215	Weight C C Fund	24	34
\$D\$216	Weight S C Fund	3	4
\$D\$217	Weight I C Fund	2.2351E-12	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	2.0000%	\$B\$211>=0	Not Binding	2.0000%
\$B\$212	Weight F	60.0000%	\$B\$212>=0	Not Binding	60.0000%
\$B\$213	Weight C	34.0000%	\$B\$213>=0	Not Binding	34.0000%
\$B\$214	Weight S	4.0000%	\$B\$214>=0	Not Binding	4.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.7999%	\$B\$216>=\$F\$213	Binding	0.0000%
\$D\$214	Weight F C Fund	60	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	34	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	4	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.7 0.8% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:47:27 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	1.8510%	2.2783%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	60	50
\$D\$215	Weight C C Fund	34	46
\$D\$216	Weight S C Fund	4	4
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	0.0000%	\$B\$211>=0	Binding	0.0000%
\$B\$212	Weight F	50.0000%	\$B\$212>=0	Not Binding	50.0000%
\$B\$213	Weight C	46.0000%	\$B\$213>=0	Not Binding	46.0000%
\$B\$214	Weight S	4.0000%	\$B\$214>=0	Not Binding	4.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.8505%	\$B\$216>=\$F\$213	Not Binding	0.0005%
\$D\$214	Weight F C Fund	50	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	46	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	4	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.8 0.85% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:48:23 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	2.2783%	2.7480%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	50	34
\$D\$215	Weight C C Fund	46	61
\$D\$216	Weight S C Fund	4	2
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	3.0000%	\$B\$211>=0	Not Binding	3.0000%
\$B\$212	Weight F	34.0000%	\$B\$212>=0	Not Binding	34.0000%
\$B\$213	Weight C	61.0000%	\$B\$213>=0	Not Binding	61.0000%
\$B\$214	Weight S	2.0000%	\$B\$214>=0	Not Binding	2.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.9000%	\$B\$216>=\$F\$213	Binding	0.0000%
\$D\$214	Weight F C Fund	34	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	61	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	2	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.9 0.9% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:50:08 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	2.7480%	3.2163%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	34	24
\$D\$215	Weight C C Fund	61	73
\$D\$216	Weight S C Fund	2	2
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	1.0000%	\$B\$211>=0	Not Binding	1.0000%
\$B\$212	Weight F	24.0000%	\$B\$212>=0	Not Binding	24.0000%
\$B\$213	Weight C	73.0000%	\$B\$213>=0	Not Binding	73.0000%
\$B\$214	Weight S	2.0000%	\$B\$214>=0	Not Binding	2.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	0.9507%	\$B\$216>=\$F\$213	Not Binding	0.0007%
\$D\$214	Weight F C Fund	24	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	73	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	2	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.10 0.95% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:50:39 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	3.2163%	3.6894%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	24	13
\$D\$215	Weight C C Fund	73	85
\$D\$216	Weight S C Fund	2	2
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	0.0000%	\$B\$211>=0	Binding	0.0000%
\$B\$212	Weight F	13.0000%	\$B\$212>=0	Not Binding	13.0000%
\$B\$213	Weight C	85.0000%	\$B\$213>=0	Not Binding	85.0000%
\$B\$214	Weight S	2.0000%	\$B\$214>=0	Not Binding	2.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	1.0002%	\$B\$216>=\$F\$213	Not Binding	0.0002%
\$D\$214	Weight F C Fund	13	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	85	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	2	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.11 1% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(post).XLS]Calcs**  
**Report Created: 2/26/2004 6:29:33 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	5.2330%	4.2059%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	0	0
\$D\$215	Weight C C Fund	1	99
\$D\$216	Weight S C Fund	99	1
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	0.0000%	\$B\$211>=0	Binding	0.0000%
\$B\$212	Weight F	0.0000%	\$B\$212>=0	Binding	0.0000%
\$B\$213	Weight C	99.0000%	\$B\$213>=0	Not Binding	99.0000%
\$B\$214	Weight S	1.0000%	\$B\$214>=0	Not Binding	1.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	1.0528%	\$B\$216>=\$F\$213	Not Binding	0.0028%
\$D\$214	Weight F C Fund	0	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	99	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	1	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.12 1.05% Return Answer Report**

**Microsoft Excel 10.0 Answer Report**  
**Worksheet: [OPTIMIZATION\_AB(17Feb).XLS]Calcs**  
**Report Created: 2/17/2004 9:52:09 PM**

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$217	Portfolio SD	3.6894%	5.2330%

Adjustable Cells

Cell	Name	Original Value	Final Value
\$D\$214	Weight F C Fund	13	0
\$D\$215	Weight C C Fund	85	1
\$D\$216	Weight S C Fund	2	99
\$D\$217	Weight I C Fund	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$211	Weight G	0.0000%	\$B\$211>=0	Binding	0.0000%
\$B\$212	Weight F	0.0000%	\$B\$212>=0	Binding	0.0000%
\$B\$213	Weight C	1.0000%	\$B\$213>=0	Not Binding	1.0000%
\$B\$214	Weight S	99.0000%	\$B\$214>=0	Not Binding	99.0000%
\$B\$215	Weight I	0.0000%	\$B\$215>=0	Binding	0.0000%
\$B\$216	Expected Return	1.0580%	\$B\$216>=\$F\$213	Binding	0.0000%
\$D\$214	Weight F C Fund	0	\$D\$214=integer	Binding	0
\$D\$215	Weight C C Fund	1	\$D\$215=integer	Binding	0
\$D\$216	Weight S C Fund	99	\$D\$216=integer	Binding	0
\$D\$217	Weight I C Fund	0	\$D\$217=integer	Binding	0

**Figure B.13 1.058% Return Answer Report**

## **Appendix C. Sample Return Probability Tables and Histograms**

**Table C.1 Active Duty Captain Probability Table**

Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13
Objective													
>=50000	1	1	1	1	1	1	1	1	1	1	1	1	1
>=100000	1	1	1	1	1	1	1	1	1	1	1	1	1
>=150000	1	1	1	1	1	1	1	1	1	1	1	0.9998	0.9984
>=200000	1	1	1	1	1	1	1	1	1	1	1	0.9994	0.9958
>=250000	1	1	1	1	1	1	1	1	1	0.9998	1	0.999	0.9914
>=300000	1	1	1	1	1	1	1	1	1	0.9992	0.9994	0.9976	0.987
>=350000	1	1	1	1	1	1	1	0.9998	0.9998	0.999	0.9984	0.9952	0.9812
>=400000	1	1	1	1	1	0.9998	0.9998	0.9998	0.9986	0.9984	0.9976	0.9928	0.9744
>=450000	1	1	1	1	1	0.9998	0.9996	0.9998	0.998	0.996	0.9954	0.9898	0.966
>=500000	1	1	1	1	0.9996	0.9998	0.9992	0.998	0.9962	0.9932	0.9908	0.9866	0.9558
>=550000	1	1	1	1	0.9986	0.9992	0.9984	0.9958	0.9924	0.991	0.9872	0.9806	0.943
>=600000	0.9998	1	0.9994	0.9992	0.9972	0.9962	0.9944	0.9942	0.9878	0.9868	0.982	0.9756	0.9316
>=650000	0.1514	0.3212	0.9858	0.9906	0.9906	0.9926	0.9908	0.9886	0.9828	0.9806	0.9768	0.967	0.9164
>=700000	0	0	0.8832	0.9656	0.976	0.9826	0.9842	0.9816	0.9764	0.9754	0.9704	0.958	0.9036
>=750000	0	0	0.585	0.9038	0.9494	0.9664	0.9742	0.9698	0.9694	0.9664	0.9628	0.9524	0.8892
>=800000	0	0	0.236	0.795	0.906	0.9452	0.9618	0.9564	0.9586	0.9572	0.954	0.9426	0.878
>=850000	0	0	0.061	0.6298	0.8454	0.916	0.9444	0.9398	0.946	0.9472	0.9446	0.933	0.8626
>=900000	0	0	0.0094	0.4564	0.7712	0.8746	0.9226	0.9236	0.9322	0.937	0.932	0.9226	0.8476
>=950000	0	0	0.001	0.3004	0.683	0.8282	0.894	0.9012	0.9178	0.9212	0.9224	0.9126	0.8346
>=1000000	0	0	0	0.177	0.581	0.776	0.8644	0.8784	0.8982	0.9104	0.908	0.8994	0.8198
>=1050000	0	0	0	0.1026	0.4856	0.7156	0.8284	0.8574	0.877	0.8952	0.8966	0.8892	0.8064
>=1100000	0	0	0	0.0556	0.3862	0.6542	0.7892	0.8284	0.8556	0.878	0.888	0.8762	0.7944
>=1150000	0	0	0	0.0282	0.2996	0.591	0.7412	0.7986	0.83	0.861	0.8726	0.8644	0.7798
>=1200000	0	0	0	0.0142	0.2376	0.5258	0.6948	0.7682	0.8096	0.8424	0.8558	0.852	0.7662
>=1250000	0	0	0	0.0054	0.176	0.4576	0.6482	0.7366	0.7898	0.8232	0.843	0.8392	0.7498
>=1300000	0	0	0	0.002	0.132	0.3954	0.6036	0.702	0.766	0.8064	0.825	0.8268	0.7376
>=1350000	0	0	0	0.001	0.0966	0.3426	0.5552	0.6676	0.742	0.7836	0.808	0.8134	0.723
>=1400000	0	0	0	0.0004	0.0666	0.2876	0.5122	0.6382	0.7148	0.7636	0.7898	0.7988	0.7126
>=1450000	0	0	0	0	0.0452	0.241	0.4686	0.6056	0.6904	0.7438	0.776	0.7856	0.6986
>=1500000	0	0	0	0	0.0314	0.2048	0.4286	0.5708	0.6664	0.7216	0.7594	0.7722	0.6862
>=1550000	0	0	0	0	0.0212	0.1752	0.3878	0.539	0.6416	0.7032	0.7434	0.7566	0.6742
>=1600000	0	0	0	0	0.0142	0.1456	0.3484	0.5112	0.6132	0.6832	0.7278	0.7452	0.6614
>=1650000	0	0	0	0	0.0102	0.1196	0.3128	0.4812	0.5878	0.6666	0.7114	0.7322	0.649
>=1700000	0	0	0	0	0.0064	0.0994	0.2806	0.452	0.563	0.6476	0.6968	0.7178	0.6384
>=1750000	0	0	0	0	0.0046	0.0836	0.2502	0.4234	0.5396	0.6264	0.681	0.705	0.6244
>=1800000	0	0	0	0	0.003	0.0704	0.2236	0.3964	0.5162	0.6106	0.6678	0.6942	0.6124
>=1850000	0	0	0	0	0.0014	0.0572	0.2	0.3714	0.4946	0.5874	0.6544	0.6806	0.601
>=1900000	0	0	0	0	0.0004	0.047	0.1762	0.3466	0.4736	0.567	0.6402	0.6684	0.5912
>=1950000	0	0	0	0	0.0002	0.039	0.1578	0.3232	0.4538	0.5488	0.6264	0.653	0.581
>=2000000	0	0	0	0	0	0.0294	0.1404	0.3016	0.4318	0.5316	0.612	0.64	0.5714
>=2050000	0	0	0	0	0	0.0238	0.1248	0.2784	0.4142	0.5144	0.599	0.6286	0.559
>=2100000	0	0	0	0	0	0.0188	0.1082	0.2598	0.397	0.4974	0.5854	0.6188	0.5496
>=2150000	0	0	0	0	0	0.016	0.0978	0.2404	0.3784	0.4812	0.5726	0.6062	0.5412
>=2200000	0	0	0	0	0	0.0124	0.0876	0.2216	0.3628	0.4646	0.5576	0.5948	0.5288
>=2250000	0	0	0	0	0	0.008	0.0766	0.2064	0.3468	0.4486	0.5442	0.583	0.5178
>=2300000	0	0	0	0	0	0.0064	0.0662	0.1898	0.3312	0.436	0.5276	0.5714	0.5078
>=2350000	0	0	0	0	0	0.0052	0.058	0.1806	0.312	0.4226	0.5176	0.5608	0.4992
>=2400000	0	0	0	0	0	0.0044	0.0496	0.167	0.2962	0.407	0.5044	0.5494	0.4898
>=2450000	0	0	0	0	0	0.0036	0.046	0.155	0.2842	0.3922	0.4872	0.5382	0.4784
>=2500000	0	0	0	0	0	0.003	0.04	0.1452	0.2672	0.3794	0.476	0.5286	0.4724
>=2550000	0	0	0	0	0	0.0026	0.0354	0.1346	0.2536	0.3642	0.4628	0.5188	0.4584
>=2600000	0	0	0	0	0	0.002	0.0298	0.126	0.24	0.3538	0.4476	0.5098	0.4502
>=2650000	0	0	0	0	0	0.0014	0.0264	0.1162	0.2296	0.341	0.4354	0.499	0.445
>=2700000	0	0	0	0	0	0.0012	0.023	0.1094	0.2174	0.3304	0.4242	0.4912	0.4386
>=2750000	0	0	0	0	0	0.0012	0.0206	0.1014	0.2062	0.318	0.4126	0.481	0.4294
>=2800000	0	0	0	0	0	0.001	0.0186	0.0944	0.1954	0.3064	0.403	0.473	0.4212
>=2850000	0	0	0	0	0	0.0006	0.016	0.0882	0.1878	0.2986	0.3924	0.465	0.416
>=2900000	0	0	0	0	0	0.0004	0.0144	0.0802	0.1762	0.2902	0.3794	0.4566	0.4104
>=2950000	0	0	0	0	0	0.0004	0.013	0.073	0.1654	0.2808	0.3678	0.445	0.4036
>=3000000	0	0	0	0	0	0.0004	0.0118	0.0678	0.1564	0.2718	0.3588	0.4362	0.3978
>=3050000	0	0	0	0	0	0.0004	0.0102	0.0638	0.1496	0.2634	0.3508	0.4268	0.3912
>=3100000	0	0	0	0	0	0.0004	0.0088	0.0594	0.1414	0.2562	0.3406	0.4182	0.3864
>=3150000	0	0	0	0	0	0.0004	0.0072	0.0548	0.133	0.2488	0.332	0.4084	0.3798
>=3200000	0	0	0	0	0	0.0004	0.006	0.0498	0.1254	0.2394	0.323	0.401	0.3732
>=3250000	0	0	0	0	0	0.0004	0.0058	0.0464	0.119	0.229	0.3144	0.3936	0.3662
>=3300000	0	0	0	0	0	0.0004	0.0044	0.0422	0.1122	0.2222	0.3066	0.3872	0.3584
>=3350000	0	0	0	0	0	0.0004	0.0042	0.039	0.1058	0.2126	0.298	0.382	0.3542
>=3400000	0	0	0	0	0	0.0004	0.0038	0.0372	0.1006	0.2062	0.2878	0.374	0.348
>=3450000	0	0	0	0	0	0.0002	0.0032	0.0354	0.0956	0.2	0.2802	0.3678	0.3426
>=3500000	0	0	0	0	0	0.0002	0.0026	0.033	0.0892	0.1926	0.272	0.3612	0.3376

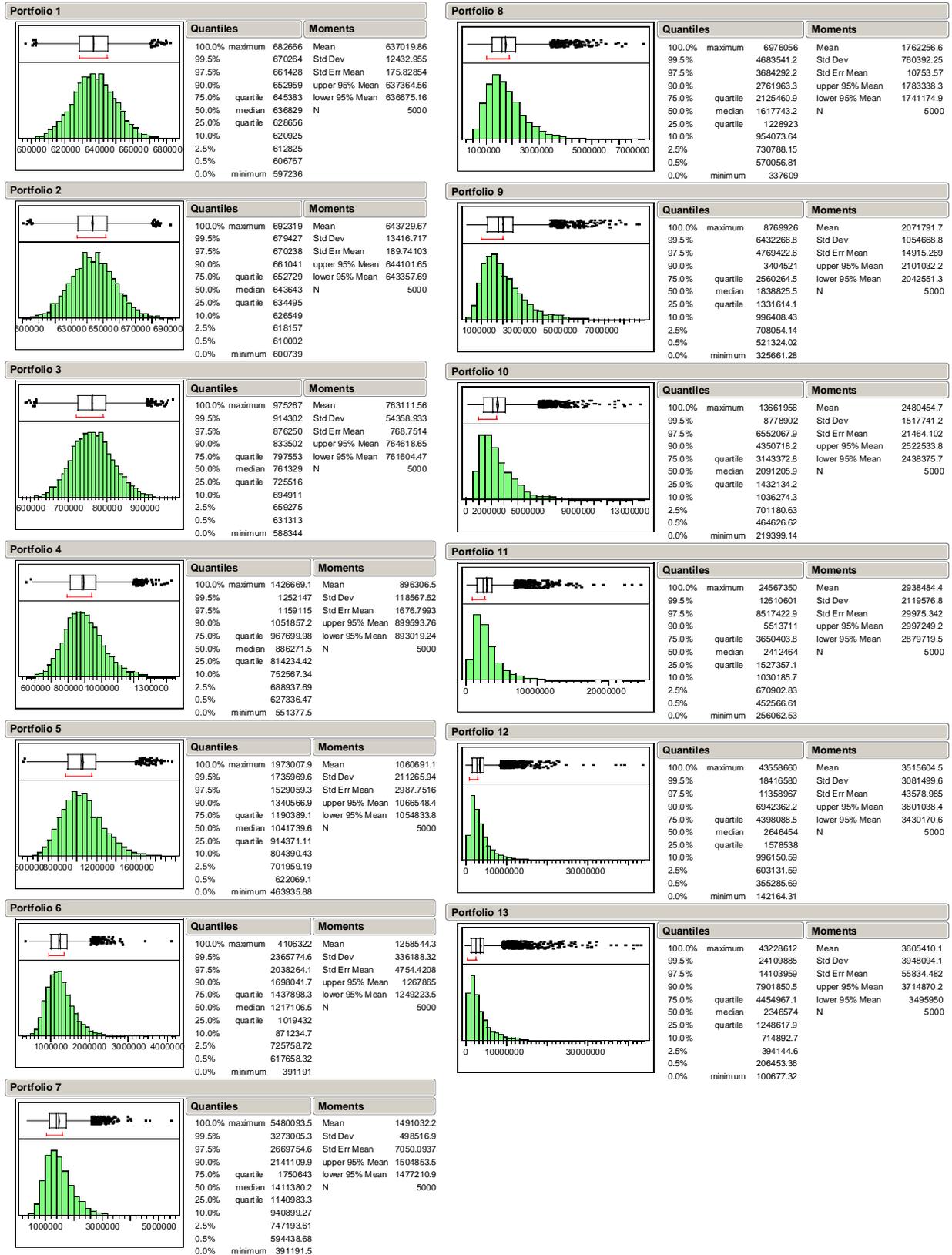


Figure C.1 Active Duty Captain Return Histograms

**Table C.2 Active Duty Lieutenant Colonel Probability Table**

Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13
Objective													
>=1000	1	1	1	1	1	1	1	1	1	1	1	1	1
>=2000	1	1	1	1	1	1	1	1	1	1	1	1	1
>=3000	1	1	1	1	1	1	1	1	1	1	1	1	1
>=4000	1	1	1	1	1	1	1	1	1	1	1	1	0.9986
>=5000	1	1	1	1	1	1	1	1	1	1	1	1	0.9978
>=6000	1	1	1	1	1	1	1	1	1	0.9998	0.9998	0.9992	0.9958
>=7000	1	1	1	1	1	1	1	1	1	0.9998	0.9994	0.9988	0.9912
>=8000	1	1	1	1	1	1	1	1	1	0.9996	0.999	0.9972	0.9868
>=9000	1	1	1	1	1	1	1	1	0.9996	0.9996	0.9984	0.9946	0.9804
>=10000	1	1	1	1	1	1	1	0.9998	0.999	0.999	0.9964	0.9924	0.972
>=11000	1	1	1	1	1	1	1	0.9998	0.9986	0.9968	0.9938	0.9882	0.9624
>=12000	1	1	1	1	1	1	0.9996	0.9996	0.9972	0.9934	0.9904	0.9834	0.948
>=13000	1	1	1	1	1	1	0.999	0.9978	0.9926	0.9904	0.9864	0.9772	0.9356
>=14000	1	1	1	1	0.9994	0.9996	0.9974	0.9952	0.9884	0.9864	0.9798	0.971	0.9238
>=15000	1	1	1	1	0.9982	0.9982	0.9946	0.9904	0.9816	0.9792	0.9722	0.9614	0.9098
>=16000	1	1	1	0.999	0.996	0.9948	0.9886	0.983	0.9758	0.9706	0.9612	0.9504	0.8964
>=17000	1	1	0.9988	0.9952	0.991	0.988	0.9804	0.9736	0.9666	0.9618	0.9522	0.9414	0.8802
>=18000	0.8832	0.941	0.9864	0.983	0.978	0.9778	0.9684	0.959	0.9524	0.9504	0.9424	0.9302	0.859
>=19000	0.017	0.0532	0.8972	0.9424	0.9476	0.9576	0.9524	0.9388	0.9366	0.9358	0.9298	0.9162	0.8428
>=20000	0	0	0.6442	0.8622	0.9076	0.9272	0.929	0.921	0.9196	0.915	0.9114	0.9026	0.8248
>=21000	0	0	0.318	0.7434	0.8482	0.8904	0.9028	0.8982	0.8996	0.899	0.895	0.8896	0.8078
>=22000	0	0	0.1082	0.5792	0.7702	0.8448	0.8686	0.8714	0.879	0.8788	0.878	0.8732	0.7846
>=23000	0	0	0.0198	0.4148	0.6796	0.7908	0.8354	0.845	0.8574	0.8576	0.86	0.8582	0.768
>=24000	0	0	0.0028	0.2782	0.57	0.7314	0.7894	0.8102	0.8314	0.8368	0.8424	0.8428	0.751
>=25000	0	0	0.0006	0.165	0.466	0.6656	0.7408	0.7732	0.803	0.8122	0.8226	0.8266	0.731
>=26000	0	0	0	0.0886	0.3696	0.5908	0.6836	0.7388	0.7764	0.7878	0.7998	0.807	0.7132
>=27000	0	0	0	0.0416	0.281	0.5188	0.629	0.6974	0.743	0.7654	0.779	0.7882	0.6928
>=28000	0	0	0	0.0176	0.2076	0.454	0.5796	0.6592	0.7094	0.7392	0.7542	0.7722	0.6762
>=29000	0	0	0	0.0078	0.1512	0.384	0.5244	0.6168	0.6786	0.7136	0.7342	0.7538	0.6614
>=30000	0	0	0	0.003	0.1122	0.3248	0.472	0.5826	0.6436	0.688	0.7144	0.7354	0.6466
>=31000	0	0	0	0.0006	0.0766	0.2696	0.4274	0.5402	0.6134	0.6612	0.6924	0.717	0.6284
>=32000	0	0	0	0.0002	0.052	0.2172	0.3828	0.5042	0.5842	0.6338	0.6678	0.6956	0.6132
>=33000	0	0	0	0	0.034	0.1762	0.334	0.4628	0.5516	0.6058	0.645	0.681	0.5986
>=34000	0	0	0	0	0.0216	0.1396	0.29	0.4218	0.5212	0.5812	0.6246	0.6604	0.5848
>=35000	0	0	0	0	0.013	0.1146	0.2588	0.3854	0.4896	0.5524	0.6014	0.6444	0.567
>=36000	0	0	0	0	0.009	0.0922	0.218	0.351	0.458	0.5256	0.5784	0.6248	0.5494
>=37000	0	0	0	0	0.0056	0.072	0.1846	0.3188	0.4344	0.4998	0.5584	0.6074	0.5338
>=38000	0	0	0	0	0.0032	0.0568	0.1584	0.292	0.4042	0.4752	0.5382	0.5932	0.52
>=39000	0	0	0	0	0.0022	0.0422	0.1378	0.2652	0.3796	0.4484	0.5204	0.5764	0.5014
>=40000	0	0	0	0	0.0016	0.0326	0.1176	0.2414	0.3554	0.425	0.4984	0.56	0.4868
>=41000	0	0	0	0	0.001	0.0248	0.098	0.219	0.332	0.4058	0.4798	0.5404	0.4736
>=42000	0	0	0	0	0.0006	0.0192	0.0806	0.1976	0.3098	0.3858	0.4616	0.5262	0.4598
>=43000	0	0	0	0	0.0006	0.0132	0.0672	0.1782	0.288	0.364	0.4458	0.509	0.4484
>=44000	0	0	0	0	0.0006	0.0098	0.0574	0.1596	0.2702	0.3482	0.4262	0.4924	0.4342
>=45000	0	0	0	0	0.0004	0.0078	0.0476	0.1416	0.2512	0.3272	0.4088	0.4762	0.4232
>=46000	0	0	0	0	0.0004	0.0064	0.0402	0.127	0.2354	0.3158	0.392	0.456	0.4124
>=47000	0	0	0	0	0.0002	0.0052	0.032	0.1172	0.2202	0.2986	0.3766	0.4438	0.3994
>=48000	0	0	0	0	0	0.0044	0.0242	0.1048	0.206	0.2806	0.3632	0.43	0.3862
>=49000	0	0	0	0	0	0.004	0.0206	0.0946	0.1912	0.2614	0.3488	0.4162	0.3738
>=50000	0	0	0	0	0	0.003	0.0178	0.0868	0.1766	0.2476	0.3332	0.404	0.363
>=51000	0	0	0	0	0	0.0018	0.0156	0.078	0.1664	0.2324	0.316	0.3902	0.3546
>=52000	0	0	0	0	0	0.0012	0.0116	0.0676	0.151	0.2202	0.3026	0.3774	0.3458
>=53000	0	0	0	0	0	0.0008	0.009	0.0598	0.1416	0.2088	0.2912	0.3644	0.3368
>=54000	0	0	0	0	0	0.0004	0.0082	0.0518	0.131	0.1956	0.2792	0.3546	0.326
>=55000	0	0	0	0	0	0.0002	0.0068	0.0462	0.1194	0.1846	0.269	0.3436	0.3164
>=56000	0	0	0	0	0	0.0002	0.0054	0.0408	0.1102	0.1746	0.257	0.3334	0.3078
>=57000	0	0	0	0	0	0.0002	0.0046	0.035	0.1022	0.1644	0.2426	0.3224	0.298
>=58000	0	0	0	0	0	0.0002	0.0038	0.0308	0.095	0.1538	0.2312	0.309	0.286
>=59000	0	0	0	0	0	0.0002	0.0034	0.0278	0.0886	0.1444	0.223	0.2982	0.2794
>=60000	0	0	0	0	0	0	0.0028	0.025	0.0804	0.1378	0.2144	0.2894	0.272
>=61000	0	0	0	0	0	0	0.0024	0.0224	0.0738	0.1294	0.205	0.2808	0.263
>=62000	0	0	0	0	0	0	0.0018	0.0206	0.0666	0.1224	0.1962	0.2716	0.2552
>=63000	0	0	0	0	0	0	0.0016	0.018	0.0604	0.115	0.1874	0.2622	0.2466
>=64000	0	0	0	0	0	0	0.0014	0.0168	0.0574	0.1082	0.1804	0.2524	0.2412
>=65000	0	0	0	0	0	0	0.0014	0.0156	0.0524	0.1024	0.172	0.2428	0.2358
>=66000	0	0	0	0	0	0	0.0014	0.0138	0.0466	0.0978	0.164	0.2354	0.2298
>=67000	0	0	0	0	0	0	0.001	0.012	0.0424	0.0922	0.159	0.2276	0.2242
>=68000	0	0	0	0	0	0	0.0006	0.0112	0.0388	0.0864	0.1512	0.219	0.2176
>=69000	0	0	0	0	0	0	0.0004	0.0094	0.0356	0.0832	0.1456	0.213	0.2128
>=70000	0	0	0	0	0	0	0.0004	0.0082	0.0334	0.078	0.139	0.2068	0.2068

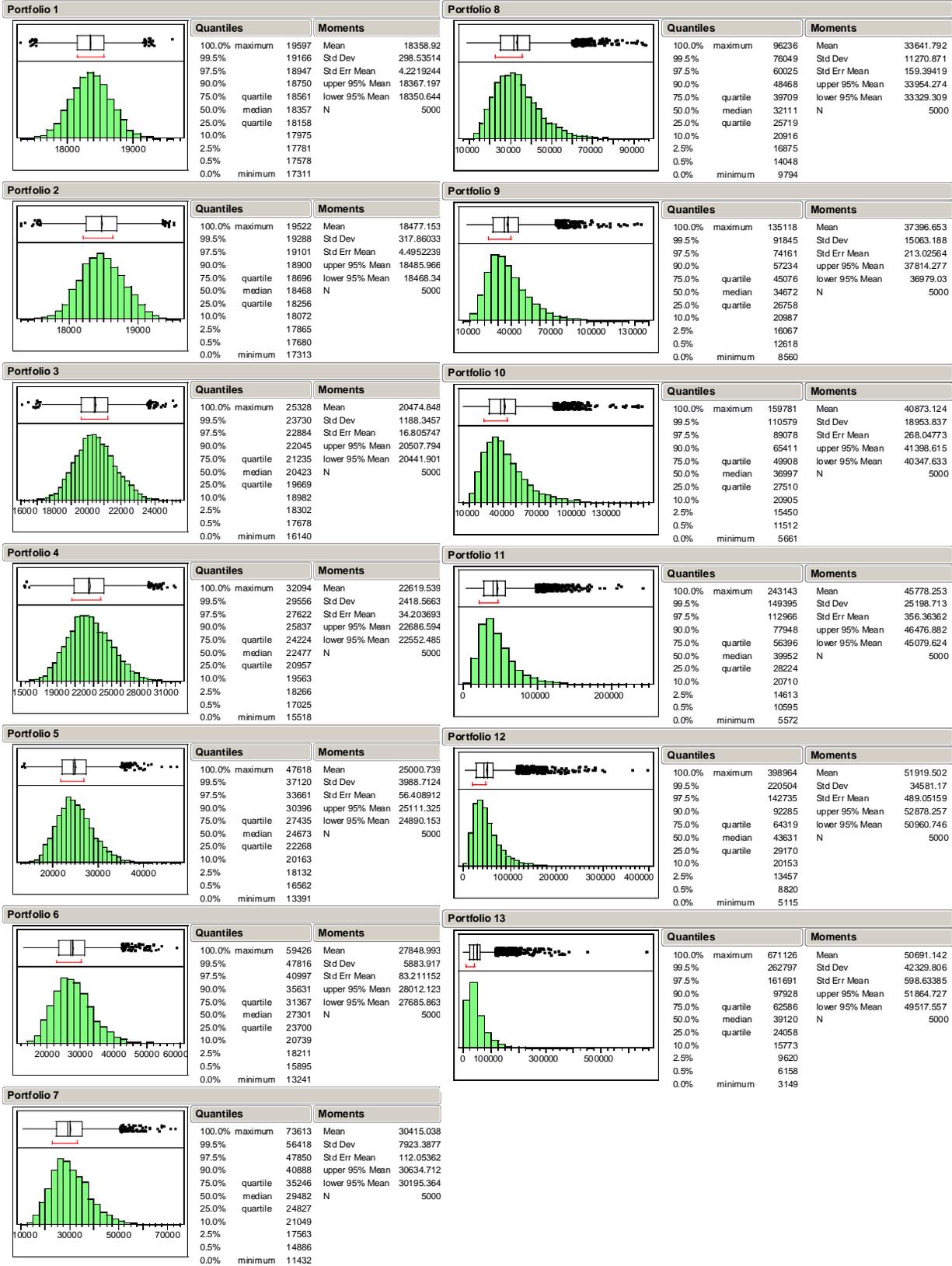


Figure C.2 Active Duty Lieutenant Colonel Return Histograms

Table C.3 GS-7 Probability Table

Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13	
Objective														
>=300000	1	1	1	1	1	1	1	1	0.9998	0.9998	0.9998	0.998	0.9858	
>=350000	1	1	1	1	1	1	1	1	0.9996	0.9992	0.999	0.9966	0.978	
>=400000	1	1	1	1	1	1	1	1	0.9994	0.9986	0.9974	0.998	0.9938	0.967
>=450000	1	1	1	1	0.9998	0.9996	0.9994	0.9982	0.9968	0.9942	0.9948	0.9902	0.9582	
>=500000	1	1	1	1	0.9996	0.9988	0.9988	0.9968	0.9946	0.9914	0.9916	0.9862	0.9492	
>=550000	1	1	0.9996	0.9994	0.998	0.997	0.9962	0.9936	0.9908	0.9866	0.9874	0.981	0.9366	
>=600000	0.1708	0.3348	0.9876	0.9926	0.9928	0.993	0.9932	0.9884	0.9856	0.9802	0.9808	0.9748	0.9228	
>=650000	0	0	0.8634	0.9604	0.978	0.9824	0.9858	0.9818	0.978	0.9716	0.9718	0.966	0.9062	
>=700000	0	0	0.5014	0.881	0.9496	0.9688	0.9732	0.9724	0.9678	0.962	0.9632	0.9584	0.8868	
>=750000	0	0	0.1582	0.7482	0.9008	0.943	0.955	0.9588	0.9536	0.9514	0.9546	0.9504	0.8718	
>=800000	0	0	0.027	0.564	0.8252	0.9	0.9318	0.9394	0.9406	0.941	0.943	0.9404	0.8574	
>=850000	0	0	0.0028	0.3758	0.7276	0.8506	0.9026	0.9196	0.926	0.9232	0.9328	0.9302	0.8436	
>=900000	0	0	0.0002	0.229	0.62	0.791	0.87	0.8946	0.9092	0.909	0.9174	0.9158	0.8268	
>=950000	0	0	0	0.1258	0.5076	0.7258	0.8252	0.8702	0.8868	0.8912	0.903	0.9004	0.8148	
>=1000000	0	0	0	0.0644	0.4032	0.6572	0.7812	0.837	0.8646	0.872	0.885	0.888	0.797	
>=1050000	0	0	0	0.0266	0.3086	0.5832	0.734	0.8054	0.8434	0.8536	0.8692	0.8734	0.7828	
>=1100000	0	0	0	0.0088	0.2274	0.512	0.6842	0.7708	0.8182	0.8362	0.8528	0.8586	0.7702	
>=1150000	0	0	0	0.0034	0.1654	0.4466	0.6288	0.729	0.7912	0.8184	0.8394	0.8458	0.7528	
>=1200000	0	0	0	0.0012	0.1186	0.3774	0.584	0.6898	0.7642	0.7962	0.8206	0.8318	0.7394	
>=1250000	0	0	0	0.0006	0.084	0.3108	0.5334	0.6512	0.733	0.7734	0.804	0.8202	0.7228	
>=1300000	0	0	0	0.0004	0.0578	0.2576	0.4826	0.6134	0.7038	0.753	0.787	0.8058	0.7074	
>=1350000	0	0	0	0.0002	0.04	0.2184	0.4314	0.5714	0.6748	0.7346	0.767	0.7904	0.694	
>=1400000	0	0	0	0.0002	0.0278	0.1782	0.387	0.536	0.6442	0.7142	0.749	0.7764	0.6796	
>=1450000	0	0	0	0.0002	0.0166	0.1426	0.3496	0.5036	0.62	0.6894	0.7316	0.7628	0.6664	
>=1500000	0	0	0	0	0.0086	0.112	0.31	0.4718	0.5914	0.6678	0.711	0.7468	0.6532	
>=1550000	0	0	0	0	0.0048	0.0888	0.2732	0.4362	0.5664	0.6474	0.6908	0.7298	0.6378	
>=1600000	0	0	0	0	0.0026	0.0686	0.24	0.4018	0.5412	0.6252	0.6738	0.7162	0.629	
>=1650000	0	0	0	0	0.0016	0.0548	0.2122	0.3714	0.5168	0.6054	0.6548	0.702	0.6156	
>=1700000	0	0	0	0	0.0014	0.043	0.1856	0.3404	0.4954	0.5834	0.6378	0.6868	0.6054	
>=1750000	0	0	0	0	0.0006	0.0342	0.1616	0.3164	0.466	0.564	0.623	0.6702	0.5942	
>=1800000	0	0	0	0	0.0004	0.0254	0.1416	0.2908	0.4434	0.5446	0.6032	0.6528	0.5812	
>=1850000	0	0	0	0	0.0002	0.0186	0.1254	0.2638	0.4228	0.5264	0.588	0.6388	0.5704	
>=1900000	0	0	0	0	0	0.0138	0.1098	0.2432	0.4016	0.5072	0.5732	0.6222	0.5574	
>=1950000	0	0	0	0	0	0.0114	0.0964	0.2218	0.3824	0.49	0.5568	0.6098	0.545	
>=2000000	0	0	0	0	0	0.0008	0.0846	0.2062	0.357	0.4698	0.5408	0.5976	0.5364	
>=2050000	0	0	0	0	0	0.0064	0.0738	0.1884	0.3382	0.4516	0.525	0.5826	0.5258	
>=2100000	0	0	0	0	0	0.0054	0.0624	0.17	0.3172	0.4358	0.511	0.5702	0.515	
>=2150000	0	0	0	0	0	0.004	0.0528	0.1562	0.303	0.4196	0.4964	0.5558	0.5046	
>=2200000	0	0	0	0	0	0.003	0.0452	0.1438	0.2892	0.404	0.4838	0.5434	0.4942	
>=2250000	0	0	0	0	0	0.0024	0.0382	0.1288	0.2746	0.3888	0.4694	0.5302	0.4846	
>=2300000	0	0	0	0	0	0.0022	0.0332	0.1162	0.26	0.3748	0.456	0.517	0.4762	
>=2350000	0	0	0	0	0	0.0016	0.0282	0.1046	0.246	0.363	0.4422	0.5056	0.4646	
>=2400000	0	0	0	0	0	0.0016	0.0238	0.0956	0.2314	0.3474	0.43	0.4956	0.4546	
>=2450000	0	0	0	0	0	0.0012	0.0204	0.0896	0.216	0.3328	0.4176	0.4858	0.4446	
>=2500000	0	0	0	0	0	0.0012	0.0166	0.08	0.204	0.3162	0.4048	0.4766	0.4316	
>=2550000	0	0	0	0	0	0.001	0.014	0.0716	0.1908	0.308	0.3934	0.4662	0.424	
>=2600000	0	0	0	0	0	0.0004	0.012	0.0662	0.181	0.2942	0.383	0.4562	0.4158	
>=2650000	0	0	0	0	0	0	0.0106	0.0624	0.17	0.2836	0.3722	0.446	0.407	
>=2700000	0	0	0	0	0	0	0.0086	0.0566	0.16	0.2752	0.3588	0.4358	0.4	
>=2750000	0	0	0	0	0	0	0.008	0.0536	0.1494	0.265	0.3488	0.4254	0.392	
>=2800000	0	0	0	0	0	0	0.0064	0.049	0.1414	0.255	0.3366	0.4148	0.3826	
>=2850000	0	0	0	0	0	0	0.0048	0.046	0.1334	0.2438	0.3268	0.4072	0.3752	
>=2900000	0	0	0	0	0	0	0.0034	0.0422	0.1266	0.2348	0.3178	0.398	0.3676	
>=2950000	0	0	0	0	0	0	0.003	0.0384	0.1178	0.2264	0.3084	0.389	0.3606	
>=3000000	0	0	0	0	0	0	0.003	0.0344	0.11	0.2192	0.3008	0.3784	0.3548	
>=3050000	0	0	0	0	0	0	0.0026	0.0316	0.1026	0.2112	0.293	0.3708	0.3488	
>=3100000	0	0	0	0	0	0	0.002	0.029	0.0952	0.2042	0.2844	0.3628	0.3434	
>=3150000	0	0	0	0	0	0	0.002	0.0266	0.0892	0.195	0.2772	0.354	0.3378	
>=3200000	0	0	0	0	0	0	0.0016	0.024	0.0842	0.1874	0.2706	0.3482	0.3316	
>=3250000	0	0	0	0	0	0	0.0014	0.0206	0.0798	0.178	0.262	0.342	0.324	
>=3300000	0	0	0	0	0	0	0.001	0.018	0.075	0.1702	0.2554	0.3368	0.3182	
>=3350000	0	0	0	0	0	0	0.0008	0.0164	0.071	0.1654	0.248	0.3306	0.3124	
>=3400000	0	0	0	0	0	0	0.0008	0.0144	0.0664	0.1584	0.241	0.3264	0.306	
>=3450000	0	0	0	0	0	0	0.0008	0.012	0.0624	0.1504	0.233	0.319	0.2996	
>=3500000	0	0	0	0	0	0	0.0008	0.0108	0.0602	0.1458	0.225	0.313	0.2934	
>=3550000	0	0	0	0	0	0	0.0006	0.0092	0.0568	0.1406	0.2182	0.306	0.2884	
>=3600000	0	0	0	0	0	0	0.0006	0.0086	0.0538	0.1334	0.2114	0.2988	0.2824	
>=3650000	0	0	0	0	0	0	0.0006	0.0084	0.0502	0.126	0.2058	0.2912	0.2746	
>=3700000	0	0	0	0	0	0	0.0006	0.007	0.0474	0.1224	0.1998	0.2852	0.2704	
>=3750000	0	0	0	0	0	0	0.0006	0.0068	0.0434	0.1174	0.193	0.2802	0.2652	

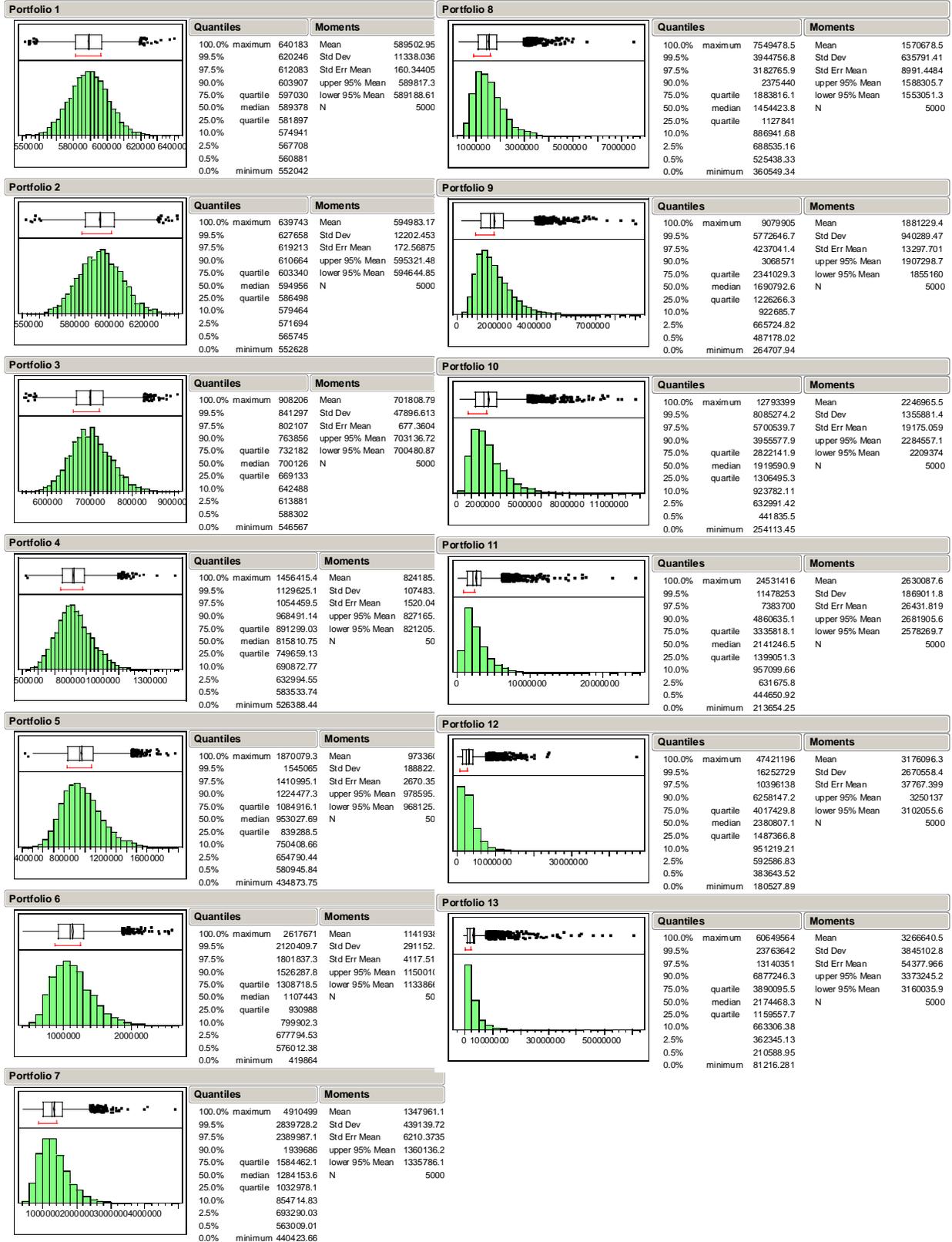


Figure C.3 GS-7 Return Histograms

Table C.4 GS-12 Probability Table

Portfolio Objective	1	2	3	4	5	6	7	8	9	10	11	12	13
>=100000	1	1	1	1	1	1	1	1	1	1	1	1	1
>=120000	1	1	1	1	1	1	1	1	1	1	1	1	0.9998
>=140000	1	1	1	1	1	1	1	1	1	1	1	1	0.9999
>=160000	1	1	1	1	1	1	1	1	1	1	1	1	0.9982
>=180000	1	1	1	1	1	1	1	1	1	1	1	0.9994	0.9964
>=200000	1	1	1	1	1	1	1	1	1	1	0.9996	0.9994	0.9934
>=220000	1	1	1	1	1	1	1	1	1	0.9998	0.9996	0.999	0.99
>=240000	1	1	1	1	1	1	1	1	1	0.9998	0.999	0.9984	0.9856
>=260000	1	1	1	1	1	1	1	1	1	0.9996	0.9982	0.9976	0.9806
>=280000	1	1	1	1	1	1	1	1	0.9998	0.9994	0.9968	0.996	0.9728
>=300000	1	1	1	1	1	1	1	0.9998	0.9996	0.9984	0.9956	0.9932	0.9674
>=320000	1	1	1	1	1	1	1	0.9994	0.9988	0.997	0.993	0.9896	0.9616
>=340000	1	1	1	1	1	1	1	0.999	0.9984	0.9964	0.9886	0.9854	0.954
>=360000	1	1	1	1	1	1	0.9998	0.9988	0.998	0.9954	0.9846	0.9814	0.9476
>=380000	1	1	1	1	1	1	0.9996	0.9974	0.9966	0.9932	0.9804	0.9764	0.9382
>=400000	1	1	1	1	1	0.9998	0.9992	0.9964	0.994	0.9902	0.9756	0.9712	0.9228
>=420000	1	1	1	1	1	0.9996	0.9974	0.9938	0.9896	0.9866	0.9692	0.9672	0.9102
>=440000	1	1	1	1	0.9996	0.9988	0.9956	0.9908	0.9868	0.9814	0.9628	0.9586	0.8976
>=460000	1	1	1	0.9998	0.9988	0.9966	0.9902	0.987	0.9804	0.9748	0.957	0.9512	0.886
>=480000	1	1	1	0.999	0.9968	0.9944	0.9844	0.9832	0.9742	0.9656	0.9504	0.9438	0.874
>=500000	1	1	0.9998	0.997	0.9932	0.9898	0.977	0.9758	0.9674	0.957	0.9416	0.9354	0.8608
>=520000	0.9996	0.9996	0.9982	0.991	0.9836	0.9798	0.9702	0.9672	0.9562	0.9476	0.9326	0.9268	0.8446
>=540000	0.765	0.8498	0.974	0.9706	0.9656	0.9682	0.9608	0.9562	0.9454	0.938	0.921	0.9158	0.8336
>=560000	0.037	0.0838	0.8868	0.9358	0.9432	0.951	0.9426	0.9424	0.9324	0.9268	0.9094	0.9056	0.8212
>=580000	0	0	0.6976	0.872	0.9122	0.9294	0.9238	0.927	0.9182	0.9148	0.8984	0.8926	0.8052
>=600000	0	0	0.4494	0.7778	0.8618	0.8988	0.8982	0.9086	0.9	0.899	0.8842	0.8808	0.792
>=620000	0	0	0.2186	0.6562	0.8072	0.864	0.875	0.8856	0.8842	0.8834	0.8714	0.8694	0.7792
>=640000	0	0	0.079	0.5276	0.7322	0.82	0.844	0.861	0.863	0.8676	0.8554	0.8548	0.7644
>=660000	0	0	0.0226	0.3974	0.6518	0.7648	0.8114	0.8328	0.8464	0.8498	0.8394	0.842	0.7496
>=680000	0	0	0.0042	0.2908	0.5702	0.716	0.7704	0.8032	0.8242	0.8316	0.824	0.826	0.7346
>=700000	0	0	0.0006	0.1914	0.4866	0.6666	0.7352	0.7766	0.8024	0.809	0.8078	0.813	0.7214
>=720000	0	0	0	0.1176	0.3998	0.6066	0.688	0.7436	0.7776	0.7882	0.7898	0.7984	0.7074
>=740000	0	0	0	0.074	0.328	0.5502	0.6452	0.7102	0.7536	0.7658	0.77	0.7832	0.6912
>=760000	0	0	0	0.0418	0.257	0.4924	0.6022	0.678	0.7268	0.7456	0.754	0.7664	0.6764
>=780000	0	0	0	0.0212	0.201	0.4372	0.5574	0.6458	0.7014	0.7212	0.736	0.7534	0.6646
>=800000	0	0	0	0.0112	0.156	0.388	0.5146	0.6116	0.6754	0.7018	0.7142	0.735	0.651
>=820000	0	0	0	0.0054	0.113	0.337	0.474	0.5752	0.6544	0.6786	0.6964	0.7212	0.6368
>=840000	0	0	0	0.0018	0.0842	0.2898	0.436	0.5436	0.6256	0.6564	0.6798	0.704	0.623
>=860000	0	0	0	0	0.0614	0.2494	0.3962	0.513	0.5986	0.633	0.6624	0.6896	0.6082
>=880000	0	0	0	0	0.0442	0.2078	0.3578	0.4786	0.5724	0.6118	0.6406	0.6706	0.5914
>=900000	0	0	0	0	0.0296	0.1728	0.3178	0.4496	0.5506	0.5902	0.622	0.6572	0.5786
>=920000	0	0	0	0	0.0194	0.1434	0.286	0.416	0.523	0.5712	0.6026	0.644	0.5608
>=940000	0	0	0	0	0.0138	0.1192	0.2526	0.3848	0.4934	0.5516	0.5842	0.6292	0.5492
>=960000	0	0	0	0	0.0102	0.0978	0.2224	0.354	0.4698	0.53	0.568	0.6118	0.5378
>=980000	0	0	0	0	0.0074	0.0782	0.1986	0.3276	0.4402	0.5118	0.5514	0.5974	0.528
>=1000000	0	0	0	0	0.005	0.064	0.1814	0.3002	0.42	0.4894	0.5342	0.5806	0.515
>=1020000	0	0	0	0	0.0026	0.0546	0.1586	0.2778	0.3974	0.4676	0.5168	0.5652	0.501
>=1040000	0	0	0	0	0.0018	0.0424	0.1364	0.257	0.3702	0.4528	0.4996	0.5496	0.4874
>=1060000	0	0	0	0	0.001	0.0334	0.1162	0.2356	0.3508	0.4304	0.4828	0.5332	0.4772
>=1080000	0	0	0	0	0.0008	0.0246	0.1032	0.2154	0.3302	0.413	0.4682	0.5208	0.4678
>=1100000	0	0	0	0	0.0006	0.0198	0.0928	0.1974	0.3112	0.3936	0.4484	0.508	0.457
>=1120000	0	0	0	0	0.0004	0.0162	0.0814	0.1842	0.293	0.376	0.4322	0.4934	0.4472
>=1140000	0	0	0	0	0	0.0128	0.0718	0.1694	0.2762	0.3624	0.4162	0.481	0.4362
>=1160000	0	0	0	0	0	0.0106	0.0646	0.1536	0.26	0.345	0.4024	0.4702	0.4262
>=1180000	0	0	0	0	0	0.008	0.0556	0.1394	0.2438	0.3272	0.3892	0.4582	0.4148
>=1200000	0	0	0	0	0	0.0054	0.0484	0.1278	0.2288	0.3114	0.376	0.4422	0.4046
>=1220000	0	0	0	0	0	0.0038	0.0406	0.1148	0.2146	0.2982	0.361	0.4276	0.397
>=1240000	0	0	0	0	0	0.0024	0.0366	0.103	0.1984	0.283	0.3494	0.4156	0.3882
>=1260000	0	0	0	0	0	0.0016	0.031	0.0916	0.1842	0.2696	0.3368	0.4038	0.3766
>=1280000	0	0	0	0	0	0.0016	0.0268	0.0848	0.1724	0.2548	0.3246	0.391	0.3692
>=1300000	0	0	0	0	0	0.0014	0.0228	0.0786	0.1606	0.2422	0.3112	0.3802	0.3618
>=1320000	0	0	0	0	0	0.001	0.0202	0.0712	0.1496	0.2334	0.2996	0.3706	0.354
>=1340000	0	0	0	0	0	0.0008	0.0154	0.0656	0.1386	0.2228	0.2874	0.3588	0.3454
>=1360000	0	0	0	0	0	0.0008	0.0136	0.0604	0.1304	0.2124	0.279	0.3484	0.3364
>=1380000	0	0	0	0	0	0.0006	0.0116	0.056	0.1218	0.2022	0.2716	0.338	0.3284
>=1400000	0	0	0	0	0	0.0004	0.0098	0.0508	0.1136	0.1904	0.2602	0.3304	0.3232
>=1420000	0	0	0	0	0	0	0.0074	0.0464	0.1034	0.1804	0.2504	0.3224	0.3142
>=1440000	0	0	0	0	0	0	0.0062	0.04	0.0968	0.1726	0.2418	0.3168	0.305
>=1460000	0	0	0	0	0	0	0.0054	0.0372	0.0912	0.1636	0.2292	0.3066	0.2968
>=1480000	0	0	0	0	0	0	0.0046	0.0328	0.0848	0.1538	0.2178	0.2988	0.291
>=1500000	0	0	0	0	0	0	0.0038	0.03	0.0782	0.1436	0.2106	0.2912	0.2822

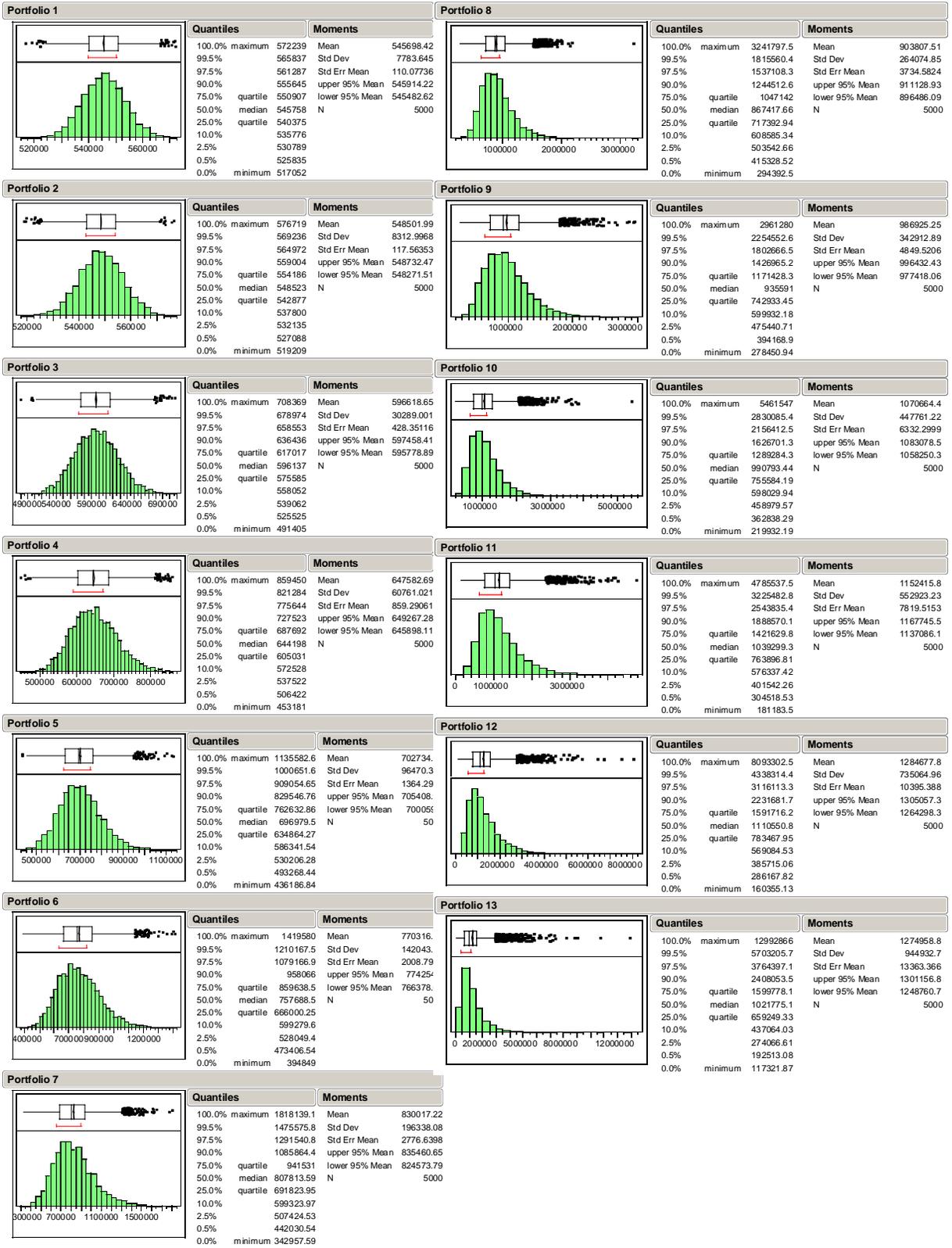


Figure C.4 GS-12 Return Histograms

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<b>1. REPORT DATE (DD-MM-YYYY)</b> 23-03-2004		<b>2. REPORT TYPE</b> Master's Thesis		<b>3. DATES COVERED (From - To)</b> Jun 2003 - Mar 2004	
<b>4. TITLE AND SUBTITLE</b>  A DECISION SUPPORT TOOL FOR THRIFT SAVINGS PLAN INVESTORS			<b>5a. CONTRACT NUMBER</b>		
			<b>5b. GRANT NUMBER</b>		
			<b>5c. PROGRAM ELEMENT NUMBER</b>		
<b>6. AUTHOR(S)</b>  Blanchette, Christopher, J., Captain, USAF			<b>5d. PROJECT NUMBER</b>		
			<b>5e. TASK NUMBER</b>		
			<b>5f. WORK UNIT NUMBER</b>		
<b>7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)</b> Air Force Institute of Technology Graduate School of Engineering and Management (AFIT/EN) 2950 Hobson Way, Building 641 WPAFB OH 45433-7765			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  AFIT/GEM/ENV/04M-02		
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> N/A			<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b>		
			<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b>		
<b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b>  APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.					
<b>13. SUPPLEMENTARY NOTES</b>					
<b>14. ABSTRACT</b> The Thrift Savings Plan is the defined contribution retirement plan for federal government employees. It is one of the largest retirement plans in the United States. The plan offers five different investment options: two funds concentrate on government securities and corporate bonds, two funds span the United States stock market, and one fund focuses on international stocks. These funds give investors the opportunity to diversify among a wide range of securities. This thesis examines the funds offered by the plan and creates a portfolio selection tool that uses investor inputs. The tool uses three mathematical models: optimization, based on Markowitz's (1952) Modern Portfolio Theory; simulation, based on the combination of optimization results and investor profiles; and selection, based on the simulation results and investor preferences. Results are presented for four investors. Other investors can use these results to gain insight, but the greatest benefit is derived from individual implementation. The tool requires only a few user inputs, and it can be operated without any external assistance. The research has further found that one investment option should not be part of any portfolio, and some portfolios can be risky to individuals for differing reasons.					
<b>15. SUBJECT TERMS</b> Thrift Savings Plan (TSP), investing, decision support, optimization, simulation, portfolio, Markowitz					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>	<b>18. NUMBER OF PAGES</b>	<b>19a. NAME OF RESPONSIBLE PERSON</b>
a. REPORT	b. ABSTRACT	c. THIS PAGE			<b>19b. TELEPHONE NUMBER (Include area code)</b>
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