TECHNICAL REPORT 2004-001

SENSITIVITY OF TRACK VELOCITY ERROR TO DATA REGISTRATION AND DYNAMIC ERRORS WITHIN A DISTRIBUTED SYSTEM

13 April 2004

JOINT SINGLE INTEGRATED AIR PICTURE (SIAP) SYSTEM ENGINEERING ORGANIZATION (JSSEO)

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7.2.6.4_TR(4-001)_1.0_JSSEO_040413

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FOREWORD

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This Technical Report is the result of the collaborative efforts of members of the technical staff at Northrop Grumman Information Technology (IT) and reviewers from the Joint SIAP System Engineering Organization (JSSEO). The following Northrop Grumman IT staff contributed to this report:

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EXECUTIVE SUMMARY

PROBLEM

The success of accurately tracking an aerospace object is strongly dependent on how well the position, velocity, and acceleration of the object is known. Understanding what contributes to errors in these quantities, and how, is important in predicting the performance of a tracking system, or as in the case of a Single Integrated Air Picture (SIAP), a distributed tracking system. A previous JSSEO Technical Report [1] investigated position estimate errors as a function of data registration and dynamic errors. This Technical Report is a complement to that earlier analysis and outlines a similar methodology for understanding and investigating track velocity estimate errors. The study of velocity estimate errors is relevant not only to the general problem of aerospace object tracking, but also to the analysis of specific operational threads. For example, in an Integrated Fire Control (IFC) engagement, the interceptor will use positional and velocity information about the target. Therefore, knowledge of the errors in both the position and velocity estimates is important to predicting system performance.

OBJECTIVES

This Technical Report presents an analytical framework for quantifying track velocity errors as a function of data registration and dynamic errors. The methodology is general and can be applied to land-based, sea-based, and airborne sensor systems. The technique provides a straightforward means of identifying the major elements that contribute to error in track velocity estimates and in quantifying their contribution to track velocity error.

APPROACH

The basis of the analysis is to develop the expression that relates the velocity of the aerospace object expressed in a sensor frame to the velocity of the object relative to the Earth's center expressed in the WGS 84 coordinate frame, and to consider how measurement, navigation, alignment, tracker, and timing (*e.g.*, time stamp, time latency, time synchronization, etc.) errors affect the estimate of track velocity. A linear error analysis is used to characterize and understand the track velocity estimation performance of an arbitrary sensor system. The process involves identifying all elements that introduce error, tracing the propagation of these errors throughout the system, and quantifying their effect on system performance. To make the analysis more tractable, certain reasonable assumptions are made. Finally, a nominal tracking scenario is used to illustrate the magnitude of the track velocity estimate error and identify which system errors are the major contributors to the total track velocity estimate error.

FINDINGS

The methodology previously used to quantify track position estimate error can be straightforwardly extended to the investigation of track velocity estimate error. The resulting track velocity error sensitivities are very similar in form to their track position error counterparts but contain some additional complexity. Comparing the results for velocity estimate error to those for position estimate error, we identify new contributors

for track velocity error: vehicle velocity errors and sensor rate measurement errors, *i.e.*, range rate, bearing rate, and elevation rate (if they are available). Relative to track position error, track velocity error is much less sensitive to the individual system errors.

CONCLUSIONS

This Technical Report provides the details of an analytical technique to understand and quantify how errors in navigation, sensor, timing, and tracker affect distributed system performance. A key result of the analysis is the determination of the sensitivities of track velocity accuracy to navigation, sensor, tracker, and timing errors. A nominal tracking scenario is considered and both the velocity error sensitivities and the position error sensitivities are computed and examined. In general, track position error is much more sensitive to the various system error terms. For the scenario considered, the track velocity error sensitivities to tracker acceleration errors and timing errors are the greatest. For position error, the sensitivities to angular-type errors and timing errors are dominant. Some similarities exist between the velocity error sensitivities and the position error sensitivities. Specifically, the position error sensitivities to navigation position errors and sensor-measured range errors are equivalent to the velocity error sensitivities to navigation velocity errors and sensor-measured range rate errors, which are just the time rates of change of navigation position and range.

Using the velocity error sensitivities with a nominal system error budget for the chosen scenario yields a sample track velocity accuracy error budget. For this particular scenario, dynamic errors dominate the total track velocity error. These dynamic errors are composed of tracker and timing error contributions. Specifically, the tracker velocity and acceleration error contributions are the primary cause for the large tracker error contribution.

We will to incorporate the results of this analysis into the JSSEO Parametric Track Accuracy Model. This will increase the scope of the model and allow for additional studies and analysis to be conducted. For example, the results of the parametric model and the system specific error parameters can be applied to using and testing the Integrated Architecture Behavior Model (IABM) within the Joint Distributed Engineering Plant (JDEP) Technical Framework. In particular, the system specific error parameters define the statistical error distribution for each error term. The parameter values coupled with the corresponding distribution define the level of bias that should be introduced into every sensor that is associated with an instantiation of the IABM. The magnitude of the contribution of each error to track velocity error provides a hierarchy for the sequence of adding errors and systems into the IABM for testing. Since larger error contributors dominate smaller contributors, the errors with smaller contributions should be introduced into the IABM first when only one system is being considered. A similar hierarchy should also be applied to testing the IABM within a distributed system. The more accurate systems should be introduced into the distributed system before the less accurate systems. The results of the error model analysis also provide examples of track velocity error that the IABM can be checked against.

RECOMMENDATIONS

The results indicate that track velocity error contributors are less important than track position error contributors in meeting distributed system capability requirements. Therefore, the IABM development activities should focus on capturing the position error contributors for the Configuration 05 build.

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1 INTRODUCTION

1.1 Problem Statement

In a previous report [1], a methodology was developed to investigate and identify the major data registration and timing elements that contribute to error in track positional estimates. The analysis contained in that report provides the basis for the ongoing JSSEO Parametric Track Accuracy Model development. This Technical Report extends that analysis to include the impact of system data registration, timing, and tracker errors on track velocity estimates. Analogous to the investigation of track position error, the methodology provides useful insight into how the various elements contribute to track velocity estimates. In addition, the results from this analysis of track velocity error can be incorporated into the Parametric Track Accuracy Model thereby enhancing its scope.

1.2 Motivation

The exchange of track positional and velocity information is critical to successfully integrating air pictures from various tracking systems. Of equal importance is the exchange of the uncertainties in those track position and velocity estimates. For a peer-to-peer environment, the exchange of track data and its associated uncertainty between the participants will be critical to the distributed system performance. It is also important to understand the origin and behavior of the uncertainty such that system performance can be analyzed and improvements identified.

In coordinating the participating systems in a network-centric environment such as Link-16, a Track Quality (TQ) Number is used to quantify track position uncertainty by reducing a three-dimensional position error covariance to a single number that is transmitted along with the track data [2]. Currently, only horizontal position errors enter into the calculation of TQ, however, future improvements may lead to additional information contributing to the TQ Number. For instance, the tracking algorithms used by the participating systems will often include the three components of velocity in their state vectors. Covariance information for the velocity error could therefore be very easily included in the calculation of TQ. An understanding of what contributes to track velocity error is necessary to determine how Track Quality would be affected by the addition of velocity error information. Such knowledge also enables investigations into other relevant areas, *e.g.*, tracking algorithms.

The study of velocity estimate errors is relevant not only to the general problem of aerospace object tracking, but also to the analysis of specific operational threads. For example, in an Integrated Fire Control (IFC) engagement, the interceptor will use positional and velocity information about the target. Therefore, knowledge of the errors in both the position and velocity estimates is important to predicting system performance.

1.3 Technical Approach

This report largely follows the same structure as the earlier report on track position error analysis [1]. Therefore, some familiarity with the content of that report is assumed to reduce redundancy here. However, Appendix A contains the position error analysis results necessary for the present treatment of velocity error. The basis of the analysis is to develop the expression that relates the velocity of the aerospace object expressed in a sensor frame to the velocity of the object relative to the Earth's center expressed in the WGS 84 coordinate frame, and to consider how measurement, navigation, alignment, timing (*e.g.*, time stamp, time latency, time synchronization, etc.), and tracker errors affect the estimate of track velocity.

The analysis is general and can be applied to land-based, sea-based, and airborne sensor systems. As with studying the position error, a linear error analysis is used to characterize and understand the track velocity estimation performance of an arbitrary sensor system. The process involves identifying all elements that introduce error, tracing the propagation of these errors throughout the system, and quantifying their effect on system performance. In the analysis, the system errors are assumed to be normal random variables represented by a deterministic mean and a variance. This type of analysis has many benefits, some of which include: quantifying how individual subsystem (*e.g.*, navigation) as well as individual elements (*e.g.*, latitude error) contribute to overall system performance, identifying potential improvement areas, predicting system performance, and enabling trade-off studies.

2 VELOCITY ERROR ANALYSIS FOR AEROSPACE OBJECTS

2.1 The Aerospace Object Velocity Vector Processing Chain

In the following series of steps, the nominal (error-free) aerospace object velocity vector processing chain is presented. The starting point of the processing chain is the sensor measurement of the object: range ρ, bearing ψ_s, and elevation θ_s. The ending point will be the object velocity vector expressed in Earth coordinates. The formulation follows directly from the position analysis (see Appendix A).

Step 1: The sensor system measures the sensor–object displacement vector in sensor coordinates,

$$(\Delta\vec{r}_{st})_s = \rho\begin{bmatrix} C_{\theta_s}C_{\psi_s} \\ C_{\theta_s}S_{\psi_s} \\ -S_{\theta_s} \end{bmatrix}_s , \tag{2.1}$$

where the notation $(\ )_s$ or $[\ ]_s$ represents the vector expressed in the s-coordinate frame and C_α and S_α are shorthand for $\cos\alpha$ and $\sin\alpha$. Differentiating $(\Delta\vec{r}_{st})_s$ with respect to time yields the time rate of change of the sensor–object displacement vector (in the sensor frame),

$$(\Delta\dot{\vec{r}}_{st})_s = \begin{bmatrix} \dot{\rho}C_{\psi_s}C_{\theta_s} - \rho\dot{\psi}_s C_{\theta_s}S_{\psi_s} - \rho\dot{\theta}_s C_{\psi_s}S_{\theta_s} \\ \dot{\rho}S_{\psi_s}C_{\theta_s} - \rho\dot{\theta}_s S_{\theta_s}S_{\psi_s} + \rho\dot{\psi}_s C_{\psi_s}C_{\theta_s} \\ -\dot{\rho}S_{\theta_s} - \rho\dot{\theta}_s C_{\theta_s} \end{bmatrix}_s . \tag{2.2}$$

The term $\dot{\rho}$ is the range rate, the term $\dot{\theta}_s$ is the elevation time rate of change, and the term $\dot{\psi}_s$ is the bearing time rate of change. While a sensor may provide a direct measure of the range rate, direct sensor measurements of the angular rates of change are very unlikely. Therefore, if we only consider the role of range rate, $(\Delta\dot{\vec{r}}_{st})_s$ reduces to

$$(\Delta\dot{\vec{r}}_{st})_s = \dot{\rho}\begin{bmatrix} C_{\psi_s}C_{\theta_s} \\ S_{\psi_s}C_{\theta_s} \\ -S_{\theta_s} \end{bmatrix}_s , \tag{2.3}$$

which mimics the expression above for $(\Delta\vec{r}_{st})_s$.

Step 2: For position, the sensor–object displacement vector is transformed to body coordinates using the sensor–body rotation matrix, R_{bs},

$$(\Delta\vec{r}_{st})_b = R_{bs}(\Delta\vec{r}_{st})_s. \tag{2.4}$$

Taking the time derivative of this expression yields the time rate of change of the sensor–object vector (in the body frame):

$$(\Delta\dot{\vec{r}}_{st})_b = R_{bs}(\Delta\dot{\vec{r}}_{st})_s + \dot{R}_{bs}(\Delta\vec{r}_{st})_s. \tag{2.5}$$

The term R_{bs} is known from the position analysis (Equation A.5), and the terms $(\Delta\vec{r}_{st})_s$ and $(\Delta\dot{\vec{r}}_{st})_s$ are given in Step 1 (Equations 2.1 and 2.2). The new term for the velocity analysis is $\dot{R}_{bs}$, which is the time rate of change of the sensor–body rotation matrix. If we make the simplifying assumption that the sensor frame is not rotating relative to the body frame, then $\dot{R}_{bs} = 0$ and the above expression reduces to

$$(\Delta\dot{\vec{r}}_{st})_b = R_{bs}(\Delta\dot{\vec{r}}_{st})_s. \tag{2.6}$$

This assumption is expected to be valid in general, even for systems such as rotating radars. The reason is that even though a radar may be mechanically rotating as part of its measurement operation, its measurement frame itself is stationary.

Step 3: For position, the body–sensor lever arm vector, $(\Delta\vec{r}_{bs})_b$, is added to yield the body–object displacement vector in body coordinates,

$$(\Delta\vec{r}_{bt})_b = (\Delta\vec{r}_{bs})_b + (\Delta\vec{r}_{st})_b. \tag{2.7}$$

The time derivative of this expression is simply,

$$(\Delta\dot{\vec{r}}_{bt})_b = (\Delta\dot{\vec{r}}_{bs})_b + (\Delta\dot{\vec{r}}_{st})_b. \tag{2.8}$$

If we make the assumption that the sensor is rigidly fixed to the vehicle body and not translating in any way, $(\Delta\dot{\vec{r}}_{bs})_b = 0$, and

$$(\Delta\dot{\vec{r}}_{bt})_b = (\Delta\dot{\vec{r}}_{st})_b. \tag{2.9}$$

Step 4: For position, the body–object displacement vector is transformed to local-level coordinates using the local-level–body rotation matrix, R_{bl},

$$(\Delta\vec{r}_{bt})_l = R_{bl}^T(\Delta\vec{r}_{bt})_b. \tag{2.10}$$

We take the time derivative of this expression to obtain the time rate of change of the body–object vector (in the local-level frame):

$$(\Delta\dot{\vec{r}}_{bt})_l = R_{bl}^T(\Delta\dot{\vec{r}}_{bt})_b + \dot{R}_{bl}^T(\Delta\vec{r}_{bt})_b. \tag{2.11}$$

The terms R_{bl}^T and $(\Delta\vec{r}_{bt})_b$ are known from the position analysis (Equations A.9 and A.7), and the term $(\Delta\dot{\vec{r}}_{bt})_b$ is given by Equation 2.8 in Step 3. The new term for the velocity analysis is $\dot{R}_{bl}^T$, which is the time rate of change of the local-level–body rotation matrix. If we make the simplifying assumption that the vehicle body is not pitching, rolling, or yawing (or that the rates of these angular motions are sufficiently small), then $\dot{R}_{bl}^T = 0$ and the above expression simplifies to

$$(\Delta\dot{\vec{r}}_{bt})_l = R_{bl}^T(\Delta\dot{\vec{r}}_{bt})_b. \tag{2.12}$$

Step 5: For position, the body–object displacement vector is transformed to Earth coordinates using the Earth–local-level rotation matrix, R_{le},

$$(\Delta\vec{r}_{bt})_e = R_{le}^T(\Delta\vec{r}_{bt})_l. \tag{2.13}$$

We take the time derivative of this expression to obtain the time rate of change of the body–object vector (in the Earth frame):

$$(\Delta\dot{\vec{r}}_{bt})_e = R_{le}^T(\Delta\dot{\vec{r}}_{bt})_l + \dot{R}_{le}^T(\Delta\vec{r}_{bt})_l. \tag{2.14}$$

The terms R_{le}^T and $(\Delta\vec{r}_{bt})_l$ are known from the position analysis (Equations A.11 and A.8), and the term $(\Delta\dot{\vec{r}}_{bt})_l$ is given by Equation 2.11 in Step 4. The new term for the velocity analysis is $\dot{R}_{le}^T$, which is the time rate of change of the Earth–local-level rotation matrix. Here, we will not assume $\dot{R}_{le}^T$ is zero since it is a function of vehicle velocity. The Earth–local-level rotation matrix is

$$R_{le} = \begin{bmatrix} -S_\phi C_\lambda & -S_\phi S_\lambda & C_\phi \\ -S_\lambda & C_\lambda & 0 \\ -C_\phi C_\lambda & -C_\phi S_\lambda & -S_\phi \end{bmatrix}. \tag{2.15}$$

The time derivative of R_{le} is

$$\dot{R}_{le} = \begin{bmatrix} S_\phi S_\lambda \dot{\lambda} - C_\lambda C_\phi \dot{\phi} & -S_\phi C_\lambda \dot{\lambda} - S_\lambda C_\phi \dot{\phi} & -S_\phi \dot{\phi} \\ -C_\lambda \dot{\lambda} & -S_\lambda \dot{\lambda} & 0 \\ C_\phi S_\lambda \dot{\lambda} + C_\lambda S_\phi \dot{\phi} & -C_\phi C_\lambda \dot{\lambda} + S_\lambda S_\phi \dot{\phi} & -C_\phi \dot{\phi} \end{bmatrix}. \tag{2.16}$$

Entering into $\dot{R}_{le}$ are the time derivatives of vehicle latitude and longitude, $\dot{\phi}$ and $\dot{\lambda}$, which are related to the north and east velocities of the vehicle. If the vehicle is stationary, $\dot{R}_{le} = 0$.

Step 6: For position, the navigation system position vector, $(\vec{r}_b)_e$, is added to yield the object position vector in WGS 84 coordinates,

$$(\vec{r}_t)_e = (\vec{r}_b)_e + (\Delta\vec{r}_{bt})_e. \tag{2.17}$$

The time derivative of this expression is simply,

$$(\dot{\vec{r}}_t)_e = (\dot{\vec{r}}_b)_e + (\Delta\dot{\vec{r}}_{bt})_e. \tag{2.18}$$

The term $(\Delta\dot{\vec{r}}_{bt})_e$ is given by Equation 2.14 in Step 5, leaving $(\dot{\vec{r}}_b)_e$, the time rate of change of the navigation system position vector, as the new term. From the position analysis (Equation A.1),

$$(\vec{r}_b)_e = \begin{bmatrix} (r_{ew} + h) C_\phi C_\lambda \\ (r_{ew} + h) C_\phi S_\lambda \\ \left[(1 - e^2) r_{ew} + h\right] S_\phi \end{bmatrix}_e, \tag{2.19}$$

where

$$r_{ew} = a_e(1 - e^2 S_\phi^2)^{-1/2}. \tag{2.20}$$

The time derivative of $(\vec{r}_b)_e$ takes the form:

$$(\dot{\vec{r}}_b)_e = \begin{bmatrix} -(r_{ew} + h)(C_\phi S_\lambda \dot{\lambda} + C_\lambda S_\phi \dot{\phi}) + (\dot{r}_{ew} + \dot{h}) C_\phi C_\lambda \\ (r_{ew} + h)(C_\phi C_\lambda \dot{\lambda} - S_\lambda S_\phi \dot{\phi}) + (\dot{r}_{ew} + \dot{h}) C_\phi S_\lambda \\ \left[(1 - e^2) r_{ew} + h\right] C_\phi \dot{\phi} + S_\phi \left[(1 - e^2) \dot{r}_{ew} + \dot{h}\right] \end{bmatrix}_e. \tag{2.21}$$

where

$$\dot{r}_{ew} = a_e e^2 S_\phi C_\phi \dot{\phi}(1 - e^2 S_\phi^2)^{-3/2}. \tag{2.22}$$

The equality,

$$\dot{r}_{ew} C_\phi - r_{ew} S_\phi \dot{\phi} = -r_{ns} S_\phi \dot{\phi}, \tag{2.23}$$

can be used to simplify $(\dot{\vec{r}}_b)_e$:

$$(\dot{\vec{r}}_b)_e = (r_{ns} + h)\dot{\phi}\begin{bmatrix} -S_\phi C_\lambda \\ -S_\phi S_\lambda \\ C_\phi \end{bmatrix}_e + (r_{ew} + h) C_\phi \dot{\lambda}\begin{bmatrix} -S_\lambda \\ C_\lambda \\ 0 \end{bmatrix}_e + \dot{h}\begin{bmatrix} C_\phi C_\lambda \\ C_\phi S_\lambda \\ S_\phi \end{bmatrix}_e, \tag{2.24}$$

where the north–south radius of curvature is

$$r_{ns} = r_{ew}(1 - e^2)(1 - e^2 S_\phi^2)^{-1}. \tag{2.25}$$

This expression provides the vehicle velocity vector expressed in Earth coordinates as a function of the vehicle geodetic position and velocity. Notice that the expression for the vehicle velocity is of the same form as the expression for the error in the position of the vehicle (Equation A.33).

2.2 The Aerospace Object Error Chain

The same steps as in the preceding section are used to derive the measured quantities from the fundamental error-free quantities. Measured quantities will be denoted with a $\tilde{}$ and error quantities will be denoted with a preceding δ.

Step 1: Starting again with the position analysis (Equation A.13), the error in the sensor–object displacement vector in sensor coordinates is,

$$(\Delta\tilde{\vec{r}}_{st})_s = (\Delta\vec{r}_{st})_s + (\delta\vec{r}_{st})_s. \tag{2.26}$$

Differentiating $(\Delta\tilde{\vec{r}}_{st})_s$ with respect to time yields the time rate of change of the error in the sensor–object displacement vector (in the sensor frame),

$$(\delta\dot{\vec{r}}_{st})_s = (\Delta\dot{\tilde{\vec{r}}}_{st})_s - (\Delta\dot{\vec{r}}_{st})_s. \tag{2.27}$$

Following the position error analysis, $\dot{\tilde{\rho}} = \dot{\rho} + \delta\dot{\rho}$, $\dot{\tilde{\theta}}_s = \dot{\theta}_s + \delta\dot{\theta}_s$, and $\dot{\tilde{\psi}}_s = \dot{\psi}_s + \delta\dot{\psi}_s$. This yields $(\delta\dot{\vec{r}}_{st})_s$ in terms of the sensor measurement errors:

$$\begin{aligned}
(\delta\dot{\vec{r}}_{st})_s \;=\; & \delta\rho\begin{bmatrix} -\dot{\psi}_s C_{\theta_s} S_{\psi_s} - \dot{\theta}_s C_{\psi_s} S_{\theta_s} \\ \dot{\psi}_s C_{\theta_s} C_{\psi_s} - \dot{\theta}_s S_{\psi_s} S_{\theta_s} \\ -\dot{\theta}_s C_{\theta_s} \end{bmatrix} + \delta\theta_s\begin{bmatrix} -\dot{\rho} S_{\theta_s} C_{\psi_s} + \rho\dot{\psi}_s S_{\psi_s} S_{\theta_s} - \rho\dot{\theta}_s C_{\psi_s} C_{\theta_s} \\ -\dot{\rho} S_{\theta_s} S_{\psi_s} - \rho\dot{\psi}_s C_{\psi_s} S_{\theta_s} - \rho\dot{\theta}_s S_{\psi_s} C_{\theta_s} \\ -\dot{\rho} C_{\theta_s} + \rho\dot{\theta}_s S_{\theta_s} \end{bmatrix} + \\
& \delta\psi_s\begin{bmatrix} -\dot{\rho} C_{\theta_s} S_{\psi_s} - \rho\dot{\psi}_s C_{\psi_s} C_{\theta_s} + \rho\dot{\theta}_s S_{\psi_s} S_{\theta_s} \\ \dot{\rho} C_{\theta_s} C_{\psi_s} - \rho\dot{\psi}_s S_{\psi_s} C_{\theta_s} - \rho\dot{\theta}_s C_{\psi_s} S_{\theta_s} \\ 0 \end{bmatrix} + \delta\dot{\rho}\begin{bmatrix} C_{\psi_s} C_{\theta_s} \\ S_{\psi_s} C_{\theta_s} \\ -S_{\theta_s} \end{bmatrix} + \\
& -\rho\delta\dot{\theta}_s\begin{bmatrix} C_{\psi_s} S_{\theta_s} \\ S_{\psi_s} S_{\theta_s} \\ C_{\theta_s} \end{bmatrix} + \rho\delta\dot{\psi}_s\begin{bmatrix} -S_{\psi_s} C_{\theta_s} \\ C_{\psi_s} C_{\theta_s} \\ 0 \end{bmatrix}.
\end{aligned} \tag{2.28}$$

Note, the last three terms in the above expression mimic the result for $(\delta\vec{r}_{st})_s$ in the position analysis (Equation A.14). If we only consider the range rate $\dot{\rho}$ and the range rate error $\delta\dot{\rho}$, the above expression reduces to

$$(\delta\dot{\vec{r}}_{st})_s = -\dot{\rho}\delta\theta_s \begin{bmatrix} S_{\theta_s}C_{\psi_s} \\ S_{\theta_s}S_{\psi_s} \\ C_{\theta_s} \end{bmatrix} + \dot{\rho}\delta\psi_s \begin{bmatrix} -C_{\theta_s}S_{\psi_s} \\ C_{\theta_s}C_{\psi_s} \\ 0 \end{bmatrix} + \delta\dot{\rho} \begin{bmatrix} C_{\psi_s}C_{\theta_s} \\ S_{\psi_s}C_{\theta_s} \\ -S_{\theta_s} \end{bmatrix}, \tag{2.29}$$

which similarly resembles $(\delta\vec{r}_{st})_s$ (Equation A.14).

Step 2: From the position analysis (Equation A.15), the error in the sensor–object displacement vector in body coordinates is

$$(\Delta\tilde{\vec{r}}_{st})_b = (\Delta\vec{r}_{st})_b + (\delta\vec{r}_{st})_b. \tag{2.30}$$

Differentiating $(\Delta\tilde{\vec{r}}_{st})_b$ with respect to time yields the time rate of change of the error in the sensor–object displacement vector (in the body frame),

$$(\delta\dot{\vec{r}}_{st})_b = (\Delta\dot{\tilde{\vec{r}}}_{st})_b - (\Delta\dot{\vec{r}}_{st})_b. \tag{2.31}$$

The full expression for $(\delta\dot{\vec{r}}_{st})_b$ is

$$(\delta\dot{\vec{r}}_{st})_b = R_{bs}(\delta\dot{\vec{r}}_{st})_s + \delta R_{bs}(\Delta\dot{\vec{r}}_{st})_s + \delta\dot{R}_{bs}(\Delta\vec{r}_{st})_s + \dot{R}_{bs}(\delta\vec{r}_{st})_s. \tag{2.32}$$

Note, the first two terms in the above expression mimic the result for $(\delta\vec{r}_{st})_b$ in the position analysis (Equation A.16). If we invoke the earlier assumption that $\dot{R}_{bs} = 0$ and also that $\delta\dot{R}_{bs} = 0$, then this expression simplifies to

$$(\delta\dot{\vec{r}}_{st})_b = R_{bs}(\delta\dot{\vec{r}}_{st})_s + \delta R_{bs}(\Delta\dot{\vec{r}}_{st})_s. \tag{2.33}$$

The terms R_{bs} and δR_{bs} are provided by the position analysis (Equations A.5 and A.17), $(\delta\dot{\vec{r}}_{st})_s$ is given by Equation 2.28 in Step 1, and $(\Delta\dot{\vec{r}}_{st})_s$ is provided by Equation 2.2 in Step 1 of Section 2.1.

Step 3: From the position analysis (Equation A.19), the error in the body–object displacement vector in body coordinates is

$$(\Delta\tilde{\vec{r}}_{bt})_b = (\Delta\vec{r}_{bt})_b + (\delta\vec{r}_{bt})_b. \tag{2.34}$$

Differentiating $(\Delta\tilde{\vec{r}}_{bt})_b$ with respect to time yields the time rate of change of the error in the body–object displacement vector (in the body frame),

$$(\delta\dot{\vec{r}}_{bt})_b = (\Delta\dot{\tilde{\vec{r}}}_{bt})_b - (\Delta\dot{\vec{r}}_{bt})_b. \tag{2.35}$$

Since we are assuming that there is no time rate of change of the body–sensor lever arm, *i.e.*, $(\Delta\dot{\vec{r}}_{bs})_b = 0$, we will also assume $(\delta\dot{\vec{r}}_{bs})_b = 0$, giving

$$(\delta\dot{\vec{r}}_{bt})_b = (\delta\dot{\vec{r}}_{st})_b. \tag{2.36}$$

with $(\delta\dot{\vec{r}}_{st})_b$ being provided by Equation 2.32 in Step 2.

Step 4: From the position analysis (Equation A.21), the error in the body–object displacement vector in local-level coordinates is

$$(\Delta\widetilde{\vec{r}}_{bt})_l = (\Delta\vec{r}_{bt})_l + (\delta\vec{r}_{bt})_l. \tag{2.37}$$

Differentiating $(\Delta\widetilde{\vec{r}}_{bt})_l$ with respect to time yields the time rate of change of the error in the body–object displacement vector (in the local-level frame),

$$(\delta\dot{\vec{r}}_{bt})_l = (\Delta\dot{\widetilde{\vec{r}}}_{bt})_l - (\Delta\dot{\vec{r}}_{bt})_l. \tag{2.38}$$

The full expression for $(\delta\dot{\vec{r}}_{bt})_l$ takes the form

$$(\delta\dot{\vec{r}}_{bt})_l = R_{bl}^T(\delta\dot{\vec{r}}_{bt})_b + \delta R_{bl}^T(\Delta\dot{\vec{r}}_{bt})_b + \delta\dot{R}_{bl}^T(\Delta\vec{r}_{bt})_b + \dot{R}_{bl}^T(\delta\vec{r}_{bt})_b. \tag{2.39}$$

Note, the first two terms in the above expression mimic the result for $(\delta\vec{r}_{bt})_l$ in the position analysis (Equation A.22). If we invoke the earlier assumption that $\dot{R}_{bl} = 0$ and also that $\delta\dot{R}_{bl} = 0$, then this expression simplifies to

$$(\delta\dot{\vec{r}}_{bt})_l = R_{bl}^T(\delta\dot{\vec{r}}_{bt})_b + \delta R_{bl}^T(\Delta\dot{\vec{r}}_{bt})_b. \tag{2.40}$$

The terms R_{bl} and δR_{bl} are provided by the position analysis (Equations A.9 and A.23), $(\delta\dot{\vec{r}}_{bt})_b$ is given by Equation 2.35 in Step 3, and $(\Delta\dot{\vec{r}}_{bt})_b$ is provided by Equation 2.8 in Step 3 of Section 2.1.

Step 5: From the position analysis (Equation A.26), the error in the body–object displacement vector in Earth coordinates is

$$(\Delta\widetilde{\vec{r}}_{bt})_e = (\Delta\vec{r}_{bt})_e + (\delta\vec{r}_{bt})_e. \tag{2.41}$$

Differentiating $(\Delta\widetilde{\vec{r}}_{bt})_e$ with respect to time yields the time rate of change of the error in the body–object displacement vector (in the Earth frame),

$$(\delta\dot{\vec{r}}_{bt})_e = (\Delta\dot{\widetilde{\vec{r}}}_{bt})_e - (\Delta\dot{\vec{r}}_{bt})_e. \tag{2.42}$$

The full expression for $(\delta\dot{\vec{r}}_{bt})_e$ takes the form

$$(\delta\dot{\vec{r}}_{bt})_e = R_{le}^T(\delta\dot{\vec{r}}_{bt})_l + \delta R_{le}^T(\Delta\dot{\vec{r}}_{bt})_l + \delta\dot{R}_{le}^T(\Delta\vec{r}_{bt})_l + \dot{R}_{le}^T(\delta\vec{r}_{bt})_l. \tag{2.43}$$

Note, the first two terms in the above expression mimic the result for $(\delta\vec{r}_{bt})_e$ in the position analysis (Equation A.27). The terms R_{le}, δR_{le}, $(\Delta\vec{r}_{bt})_l$, and $(\delta\vec{r}_{bt})_l$ are provided by the position analysis (Equations A.11, A.28, A.8, and A.22), $(\delta\dot{\vec{r}}_{bt})_l$ is given by Equation 2.39 in Step 4, and $(\Delta\dot{\vec{r}}_{bt})_l$ is provided by Equation 2.11 in Step 4 of Section 2.1. The term $\dot{R}_{le}^T$ can be obtained from Equation 2.16 in Step 5 of Section 2.1. The term $\delta\dot{R}_{le}^T$ can be derived using the results from the position analysis (Equation A.28),

$$\delta R_{le} = \delta\Omega_{le}\widetilde{R}_{le}. \tag{2.44}$$

Taking the time derivative of this expression yields

$$\delta\dot{R}_{le} = \delta\Omega_{le}\dot{\tilde{R}}_{le} + \delta\dot{\Omega}_{le}\tilde{R}_{le}. \tag{2.45}$$

The term $\delta\Omega_{le}$ is provided in the position analysis (Equation A.29) and its time derivative is

$$\delta\dot{\Omega}_{le} = \begin{bmatrix} 0 & \delta\dot{\phi}_3 & -\delta\dot{\phi}_2 \\ -\delta\dot{\phi}_3 & 0 & \delta\dot{\phi}_1 \\ \delta\dot{\phi}_2 & -\delta\dot{\phi}_1 & 0 \end{bmatrix}. \tag{2.46}$$

Also from the position analysis (Equation A.30),

$$(\delta\vec{\phi})_l = \begin{bmatrix} \delta\phi_1 \\ \delta\phi_2 \\ \delta\phi_3 \end{bmatrix}_l = -\delta\phi\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_l + \delta\lambda\begin{bmatrix} C_\phi \\ 0 \\ -S_\phi \end{bmatrix}_l, \tag{2.47}$$

which has the time derivative,

$$(\delta\dot{\vec{\phi}})_l = \begin{bmatrix} \delta\dot{\phi}_1 \\ \delta\dot{\phi}_2 \\ \delta\dot{\phi}_3 \end{bmatrix}_l = -\dot{\phi}\delta\lambda\begin{bmatrix} S_\phi \\ 0 \\ C_\phi \end{bmatrix}_l - \delta\dot{\phi}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_l + \delta\dot{\lambda}\begin{bmatrix} C_\phi \\ 0 \\ -S_\phi \end{bmatrix}_l. \tag{2.48}$$

Note, the last two terms in the above expression mimic the result for $(\delta\vec{\phi})_l$ in the position analysis (Equation A.30).

Step 6: From the position analysis (Equation A.31), the error in the object position vector in Earth coordinates is

$$(\tilde{\vec{r}}_t)_e = (\vec{r}_t)_e + (\delta\vec{r}_t)_e. \tag{2.49}$$

Differentiating $(\tilde{\vec{r}})_e$ with respect to time yields the time rate of change of the error in the object velocity vector (in the Earth frame),

$$(\delta\dot{\vec{r}}_t)_e = (\dot{\tilde{\vec{r}}}_t)_e - (\dot{\vec{r}}_t)_e, \tag{2.50}$$

which is equal to

$$(\delta\dot{\vec{r}}_t)_e = (\delta\dot{\vec{r}}_b)_e + (\delta\dot{\vec{r}}_{bt})_e. \tag{2.51}$$

The term $(\delta\dot{\vec{r}}_{bt})_e$ is known from Equation 2.43 in Step 5 above. The term $(\delta\dot{\vec{r}}_b)_e$ is the error in the vehicle velocity vector expressed in the Earth frame, and must be determined. From the position analysis (Equation A.33), we know

$$(\delta\vec{r}_b)_e = (r_{ns} + h)\delta\phi\begin{bmatrix} -S_\phi C_\lambda \\ -S_\phi S_\lambda \\ C_\phi \end{bmatrix}_e + (r_{ew} + h)C_\phi\delta\lambda\begin{bmatrix} -S_\lambda \\ C_\lambda \\ 0 \end{bmatrix}_e + \delta h\begin{bmatrix} C_\phi C_\lambda \\ C_\phi S_\lambda \\ S_\phi \end{bmatrix}_e. \tag{2.52}$$

The time derivative of $(\delta\vec{r}_b)_e$ yields

$$\begin{aligned}(\dot{\delta\vec{r}_b})_e &= \delta\phi\begin{bmatrix}(r_{ns}+h)(S_\phi S_\lambda\dot{\lambda}-C_\phi C_\lambda\dot{\phi})-(\dot{r}_{ns}+\dot{h})S_\phi C_\lambda\\-(r_{ns}+h)(S_\phi C_\lambda\dot{\lambda}+C_\phi S_\lambda\dot{\phi})-(\dot{r}_{ns}+\dot{h})S_\phi S_\lambda\\-(r_{ns}+h)S_\phi\dot{\phi}+(\dot{r_{ns}}+\dot{h})C_\phi\end{bmatrix}_e+\\&\delta\lambda\begin{bmatrix}(r_{ew}+h)(-C_\phi C_\lambda\dot{\lambda}+S_\phi S_\lambda\dot{\phi})-(\dot{r}_{ew}+\dot{h})C_\phi S_\lambda\\-(r_{ew}+h)(C_\phi S_\lambda\dot{\lambda}+S_\phi C_\lambda\dot{\phi})+(\dot{r}_{ew}+\dot{h})C_\phi C_\lambda\\0\end{bmatrix}_e+\\&\delta h\begin{bmatrix}-C_\phi S_\lambda\dot{\lambda}-S_\phi C_\lambda\dot{\phi}\\C_\phi C_\lambda\dot{\lambda}-S_\phi S_\lambda\dot{\phi}\\C_\phi\dot{\phi}\end{bmatrix}_e+(r_{ns}+h)\delta\dot{\phi}\begin{bmatrix}-S_\phi C_\lambda\\-S_\phi S_\lambda\\C_\phi\end{bmatrix}_e+\\&(r_{ew}+h)C_\phi\delta\dot{\lambda}\begin{bmatrix}-S_\lambda\\C_\lambda\\0\end{bmatrix}_e+\delta\dot{h}\begin{bmatrix}C_\phi C_\lambda\\C_\phi S_\lambda\\S_\phi\end{bmatrix}_e,\end{aligned}\tag{2.53}$$

where

$$\dot{r}_{ew}=a_e e^2 S_\phi C_\phi\dot{\phi}(1-e^2S_\phi^2)^{-3/2}\tag{2.54}$$

and

$$\dot{r}_{ns}=3(1-e^2)\dot{r}_{ew}(1-e^2S_\phi^2)^{-1}.\tag{2.55}$$

When computing the velocity error sensitivities, we will want to express the sensitivity of the object velocity error to north and east position (and velocity) errors. Therefore, it is useful to express $(\dot{\delta\vec{r}_b})_e$ in terms of errors in north and east coordinates. Error in latitude ($\delta\phi$) maps to error in north position as $\delta x_N=\delta\phi(r_{ns}+h)$. Longitude error ($\delta\lambda$) maps to east position error as $\delta x_E=C_\phi\delta\lambda(r_{ew}+h)$. When these substitutions are made, the resulting expression for $(\dot{\delta\vec{r}_b})_e$ is:

$$\begin{aligned}(\dot{\delta\vec{r}_b})_e &= \delta x_N\begin{bmatrix}S_\phi S_\lambda\dot{\lambda}-C_\phi C_\lambda\dot{\phi}\\-S_\phi C_\lambda\dot{\lambda}-C_\phi S_\lambda\dot{\phi}\\-S_\phi\dot{\phi}\end{bmatrix}_e+\delta x_E\dot{\lambda}\begin{bmatrix}-C_\lambda\\-S_\lambda\\0\end{bmatrix}_e+\delta h\begin{bmatrix}-C_\phi S_\lambda\dot{\lambda}-S_\phi C_\lambda\dot{\phi}\\C_\phi C_\lambda\dot{\lambda}-S_\phi S_\lambda\dot{\phi}\\C_\phi\dot{\phi}\end{bmatrix}_e+\\&\delta v_N\begin{bmatrix}-S_\phi C_\lambda\\-S_\phi S_\lambda\\C_\phi\end{bmatrix}_e+\delta v_E\begin{bmatrix}-S_\lambda\\C_\lambda\\0\end{bmatrix}_e+\delta\dot{h}\begin{bmatrix}C_\phi C_\lambda\\C_\phi S_\lambda\\S_\phi\end{bmatrix}_e.\end{aligned}\tag{2.56}$$

2.3 Dynamic Errors

2.3.1 Tracker errors

The object velocity error incurred as a result of the tracking algorithm is given by

$$(\delta\vec{v})_e+(\delta\vec{a})_e\Delta T,\tag{2.57}$$

where ΔT is the true time latency of the system. The term $(\delta\vec{v})_e$ represents any error the tracking algorithm adds to the object velocity (*e.g.*, smoothing errors). The term $(\delta\vec{a})_e$ represents the error in the acceleration used in the tracking algorithm to extrapolate the object position. These errors are expressed in the Earth frame.

2.3.2 Timing errors

The object velocity error incurred as a result of a timing error (δt) is given by

$$(\vec{a})_e \delta t, \tag{2.58}$$

where $(\vec{a})_e$ is the object acceleration expressed in the Earth frame. Timing errors can include time stamp error, time synchronization errors (*e.g.*, host-to-network error), time latency errors, etc.

2.4 Total Aerospace Object Velocity Error

The total error in the object velocity, expressed in the Earth frame, is the sum of the previous results:

$$\left[(\delta \dot{\vec{r}}_t)_e\right]_{\text{total}} = \underbrace{(\delta \dot{\vec{r}}_t)_e}_{\text{data reg}} + \underbrace{\underbrace{(\delta \vec{v})_e + (\delta \vec{a})_e \Delta T}_{\text{tracker}} + \underbrace{(\vec{a})_e \delta t}_{\text{timing}}}_{\text{dynamic}}. \tag{2.59}$$

The total error can be partitioned into data registration errors and dynamic errors as indicated above.

3 THE VELOCITY ERROR SENSITIVITIES

With the object velocity vector completely described in Section 2.1 and the errors derived in Sections 2.2–2.4, the velocity error sensitivities can be derived. The ability to express the sensitivities explicitly is a direct result of the linear error analysis employed in Section 2. We are interested in defining the sensitivities of the object velocity error to the various individual error sources (*i.e.*, position errors, tilt errors, time synchronization errors, etc.) The total velocity error can be compactly written as

$$(\delta\dot{\vec{r}}_t)_e = S\,\vec{e}, \tag{3.1}$$

where S is a $3\times N$ sensitivity matrix (N is the number of individual system error sources considered) and $\vec{e}$ is the $N\times 1$ system error vector. This form is advantageous since the covariance of the track velocity error can be directly obtained from the expression

$$\mathrm{cov}(\delta\dot{\vec{r}}_t)_e = S\,\mathrm{cov}(\vec{e})\,S^T. \tag{3.2}$$

Similarly, the covariance of the track velocity error for groups of error contributors (*e.g.*, all of navigation) can be easily calculated by considering the appropriate submatrices of S. The elements of S provide information on "how fast" the components of track velocity error change as the individual error terms change. The three rows of S correspond to the x-, y-, and z-components of the WGS 84 coordinate system and each column of S corresponds to an element of the error vector. In the expressions below, the notation $S_{\delta\epsilon}$ represents a single column of the sensitivity matrix and denotes the track velocity error sensitivity to the error ϵ.

In what follows, the north and east velocities of the vehicle need to be converted to time rates of change of latitude and longitude:

$$\dot{\phi} = \frac{v_N}{r_{ns}+h} \tag{3.3}$$

and

$$\dot{\lambda} = \frac{v_E}{C_\phi(r_{ew}+h)}. \tag{3.4}$$

To north position error:

$$\begin{aligned} S_{\delta x_N} \; = \; & \begin{bmatrix} S_\phi S_\lambda \dot{\lambda} - C_\phi C_\lambda \dot{\phi} \\ -S_\phi C_\lambda \dot{\lambda} - C_\phi S_\lambda \dot{\phi} \\ -S_\phi \dot{\phi} \end{bmatrix}_e + \frac{R_{le}^T}{r_{ns}+h} \begin{bmatrix} -\Delta \dot{r}_{bt_{l_3}} \\ 0 \\ \Delta \dot{r}_{bt_{l_1}} \end{bmatrix}_l + \\ & \frac{\dot{R}_{le}^T}{r_{ns}+h} \begin{bmatrix} -\Delta r_{bt_{l_3}} \\ 0 \\ \Delta r_{bt_{l_1}} \end{bmatrix}_l - \frac{R_{le}^T(\dot{r}_{ns}+\dot{h})}{(r_{ns}+h)^2} \begin{bmatrix} -\Delta r_{bt_{l_3}} \\ 0 \\ \Delta r_{bt_{l_1}} \end{bmatrix}_l \end{aligned} \tag{3.5}$$

To east position error:

$$S_{\delta x_E} = \dot{\lambda}\begin{bmatrix} -C_\lambda \\ -S_\lambda \\ 0 \end{bmatrix}_e + \frac{R_{le}^T}{C_\phi(r_{ew}+h)}\begin{bmatrix} S_\phi \Delta\dot{r}_{bt_{l_2}} \\ -S_\phi \Delta\dot{r}_{bt_{l_1}} - C_\phi \Delta\dot{r}_{bt_{l_3}} \\ C_\phi \Delta\dot{r}_{bt_{l_2}} \end{bmatrix}_l +$$
$$\frac{\dot{R}_{le}^T}{C_\phi(r_{ew}+h)}\begin{bmatrix} S_\phi \Delta r_{bt_{l_2}} \\ -S_\phi \Delta r_{bt_{l_1}} - C_\phi \Delta r_{bt_{l_3}} \\ C_\phi \Delta r_{bt_{l_2}} \end{bmatrix}_l +$$
$$\frac{R_{le}^T}{C_\phi(r_{ew}+h)}\begin{bmatrix} \left(C_\phi\dot{\phi} - S_\phi\frac{\dot{r}_{ew}+\dot{h}}{r_{ew}+h} + \frac{S_\phi^2}{C_\phi}\dot{\phi}\right)\Delta r_{bt_{l_2}} \\ \left(-C_\phi\dot{\phi} + S_\phi\frac{\dot{r}_{ew}+\dot{h}}{r_{ew}+h} - \frac{S_\phi^2}{C_\phi}\dot{\phi}\right)\Delta r_{bt_{l_1}} + C_\phi\frac{\dot{r}_{ew}+\dot{h}}{r_{ew}+h}\Delta r_{bt_{l_3}} \\ -C_\phi\frac{\dot{r}_{ew}+\dot{h}}{r_{ew}+h}\Delta r_{bt_{l_2}} \end{bmatrix}_l \tag{3.6}$$

To height error:

$$S_{\delta h} = \begin{bmatrix} -C_\phi S_\lambda\dot{\lambda} - S_\phi C_\lambda\dot{\phi} \\ C_\phi C_\lambda\dot{\lambda} - S_\phi S_\lambda\dot{\phi} \\ C_\phi\dot{\phi} \end{bmatrix}_e \tag{3.7}$$

To north velocity error:

$$S_{\delta v_N} = \begin{bmatrix} -S_\phi C_\lambda \\ -S_\phi S_\lambda \\ C_\phi \end{bmatrix}_e + \frac{R_{le}^T}{r_{ns}+h}\begin{bmatrix} -\Delta r_{bt_{l_3}} \\ 0 \\ \Delta r_{bt_{l_1}} \end{bmatrix}_l \tag{3.8}$$

To east velocity error:

$$S_{\delta v_E} = \begin{bmatrix} -S_\lambda \\ C_\lambda \\ 0 \end{bmatrix}_e + \frac{R_{le}^T}{C_\phi(r_{ew}+h)}\begin{bmatrix} S_\phi \Delta r_{bt_{l_2}} \\ -S_\phi \Delta r_{bt_{l_1}} - C_\phi \Delta r_{bt_{l_3}} \\ C_\phi \Delta r_{bt_{l_2}} \end{bmatrix}_l \tag{3.9}$$

To vertical velocity error:

$$S_{\delta \dot{h}} = \begin{bmatrix} C_\phi C_\lambda \\ C_\phi S_\lambda \\ S_\phi \end{bmatrix}_e \tag{3.10}$$

To navigation tilt (north) error:

$$S_{\delta\theta_N} = R_{le}^T R_{bl}^T \begin{bmatrix} 0 \\ -\Delta\dot{r}_{bt_{b_3}} \\ \Delta\dot{r}_{bt_{b_2}} \end{bmatrix}_b + \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} 0 \\ -\Delta r_{bt_{b_3}} \\ \Delta r_{bt_{b_2}} \end{bmatrix}_b \tag{3.11}$$

To navigation tilt (east) error:

$$S_{\delta\theta_E} = R_{le}^T R_{bl}^T \begin{bmatrix} \Delta\dot{r}_{bt_{b_3}} \\ 0 \\ -\Delta\dot{r}_{bt_{b_1}} \end{bmatrix}_b + \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} \Delta r_{bt_{b_3}} \\ 0 \\ -\Delta r_{bt_{b_1}} \end{bmatrix}_b \tag{3.12}$$

To navigation tilt (down) error:

$$S_{\delta\theta_D} = R_{le}^T R_{bl}^T \begin{bmatrix} -\Delta\dot{r}_{bt_{b_2}} \\ \Delta\dot{r}_{bt_{b_1}} \\ 0 \end{bmatrix}_b + \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} -\Delta r_{bt_{b_2}} \\ \Delta r_{bt_{b_1}} \\ 0 \end{bmatrix}_b \tag{3.13}$$

To lever arm (b_1) error:

$$S_{\delta b_1} = \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_b \tag{3.14}$$

To lever arm (b_2) error:

$$S_{\delta b_2} = \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_b \tag{3.15}$$

To lever arm (b_3) error:

$$S_{\delta b_3} = \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_b \tag{3.16}$$

To sensor misalignment (ψ_1):

$$S_{\delta\psi_1} = R_{le}^T R_{bl}^T \begin{bmatrix} 0 \\ \Delta\dot{r}_{st_{b_3}} \\ -\Delta\dot{r}_{st_{b_2}} \end{bmatrix}_b + \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} 0 \\ \Delta r_{st_{b_3}} \\ -\Delta r_{st_{b_2}} \end{bmatrix}_b \tag{3.17}$$

To sensor misalignment (ψ_2):

$$S_{\delta\psi_2} = R_{le}^T R_{bl}^T \begin{bmatrix} -\Delta\dot{r}_{st_{b_3}} \\ 0 \\ \Delta\dot{r}_{st_{b_1}} \end{bmatrix}_b + \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} -\Delta r_{st_{b_3}} \\ 0 \\ \Delta r_{st_{b_1}} \end{bmatrix}_b \tag{3.18}$$

To sensor misalignment (ψ_3):

$$S_{\delta\psi_3} = R_{le}^T R_{bl}^T \begin{bmatrix} \Delta\dot{r}_{st_{b_2}} \\ -\Delta\dot{r}_{st_{b_1}} \\ 0 \end{bmatrix}_b + \dot{R}_{le}^T R_{bl}^T \begin{bmatrix} \Delta r_{st_{b_2}} \\ -\Delta r_{st_{b_1}} \\ 0 \end{bmatrix}_b \tag{3.19}$$

To sensor range error:

$$S_{\delta\rho} = \dot{R}_{le}^T R_{bl}^T R_{bs} \begin{bmatrix} C_{\theta_s} C_{\psi_s} \\ C_{\theta_s} S_{\psi_s} \\ -S_{\theta_s} \end{bmatrix}_s + R_{le}^T R_{bl}^T R_{bs} \begin{bmatrix} -\dot{\psi}_s C_{\theta_s} S_{\psi_s} - \dot{\theta}_s C_{\psi_s} S_{\theta_s} \\ \dot{\psi}_s C_{\theta_s} C_{\psi_s} - \dot{\theta}_s S_{\psi_s} S_{\theta_s} \\ -\dot{\theta}_s C_{\theta_s} \end{bmatrix}_s \tag{3.20}$$

Considering only range and range rate measurements,

$$S_{\delta\rho} = \dot{R}_{le}^T R_{bl}^T R_{bs} \begin{bmatrix} C_{\theta_s} C_{\psi_s} \\ C_{\theta_s} S_{\psi_s} \\ -S_{\theta_s} \end{bmatrix}_s \tag{3.21}$$

To sensor bearing error:

$$S_{\delta\psi_s} = \dot{R}_{le}^T R_{bl}^T R_{bs} \rho \begin{bmatrix} -C_{\theta_s} S_{\psi_s} \\ C_{\theta_s} C_{\psi_s} \\ 0 \end{bmatrix}_s + R_{le}^T R_{bl}^T R_{bs} \begin{bmatrix} -\dot{\rho} C_{\theta_s} S_{\psi_s} - \rho \dot{\psi}_s C_{\psi_s} C_{\theta_s} + \rho \dot{\theta}_s S_{\psi_s} S_{\theta_s} \\ \dot{\rho} C_{\theta_s} C_{\psi_s} - \rho \dot{\psi}_s S_{\psi_s} C_{\theta_s} - \rho \dot{\theta}_s C_{\psi_s} S_{\theta_s} \\ 0 \end{bmatrix}_s \tag{3.22}$$

Considering only range and range rate measurements,

$$S_{\delta\psi_s} = \dot{R}_{le}^T R_{bl}^T R_{bs} \rho \begin{bmatrix} -C_{\theta_s} S_{\psi_s} \\ C_{\theta_s} C_{\psi_s} \\ 0 \end{bmatrix}_s + R_{le}^T R_{bl}^T R_{bs} \dot{\rho} \begin{bmatrix} -C_{\theta_s} S_{\psi_s} \\ C_{\theta_s} C_{\psi_s} \\ 0 \end{bmatrix}_s \tag{3.23}$$

To sensor elevation error:

$$S_{\delta\theta_s} = \dot{R}_{le}^T R_{bl}^T R_{bs} \rho \begin{bmatrix} -S_{\theta_s} C_{\psi_s} \\ -S_{\theta_s} S_{\psi_s} \\ -C_{\theta_s} \end{bmatrix}_s + R_{le}^T R_{bl}^T R_{bs} \begin{bmatrix} -\dot{\rho} S_{\theta_s} C_{\psi_s} + \rho \dot{\psi}_s S_{\psi_s} S_{\theta_s} - \rho \dot{\theta}_s C_{\psi_s} C_{\theta_s} \\ -\dot{\rho} S_{\theta_s} S_{\psi_s} - \rho \dot{\psi}_s C_{\psi_s} S_{\theta_s} - \rho \dot{\theta}_s S_{\psi_s} C_{\theta_s} \\ -\dot{\rho} C_{\theta_s} + \rho \dot{\theta}_s S_{\theta_s} \end{bmatrix}_s \quad (3.24)$$

Considering only range and range rate measurements,

$$S_{\delta\theta_s} = \dot{R}_{le}^T R_{bl}^T R_{bs} \rho \begin{bmatrix} -S_{\theta_s} C_{\psi_s} \\ -S_{\theta_s} S_{\psi_s} \\ -C_{\theta_s} \end{bmatrix}_s - R_{le}^T R_{bl}^T R_{bs} \dot{\rho} \begin{bmatrix} S_{\theta_s} C_{\psi_s} \\ S_{\theta_s} S_{\psi_s} \\ C_{\theta_s} \end{bmatrix}_s \quad (3.25)$$

To sensor range rate error:

$$S_{\delta\dot{\rho}} = R_{le}^T R_{bl}^T R_{bs} \begin{bmatrix} C_{\psi_s} C_{\theta_s} \\ S_{\psi_s} C_{\theta_s} \\ -S_{\theta_s} \end{bmatrix}_s \quad (3.26)$$

To sensor bearing rate error:

$$S_{\delta\dot{\psi}_s} = R_{le}^T R_{bl}^T R_{bs} \rho \begin{bmatrix} -S_{\psi_s} C_{\theta_s} \\ C_{\psi_s} C_{\theta_s} \\ 0 \end{bmatrix}_s \quad (3.27)$$

To sensor elevation rate error:

$$S_{\delta\dot{\theta}_s} = R_{le}^T R_{bl}^T R_{bs} \rho \begin{bmatrix} -C_{\psi_s} S_{\theta_s} \\ -S_{\psi_s} S_{\theta_s} \\ -C_{\theta_s} \end{bmatrix}_s \quad (3.28)$$

To tracker position errors:

$$S_{\delta x} = S_{\delta y} = S_{\delta z} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.29)$$

To tracker x-velocity error:

$$S_{\delta v_x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3.30}$$

To tracker y-velocity error:

$$S_{\delta v_y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{3.31}$$

To tracker z-velocity error:

$$S_{\delta v_z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.32}$$

To tracker x-acceleration error:

$$S_{\delta a_x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Delta T \tag{3.33}$$

To tracker y-acceleration error:

$$S_{\delta a_y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Delta T \tag{3.34}$$

To tracker z-acceleration error:

$$S_{\delta a_z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Delta T \tag{3.35}$$

To timing errors:

$$S_{\delta t} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_e \tag{3.36}$$

4 RESULTS

4.1 Scenario Description

To develop some physical intuition, we will consider a simple tracking case to identify the important sensitivities and quantify the relative importance of the error contributors. These results can be compared to the results for position error for additional insight. The scenario (see Figure 4.1) consists of a sea-based sensor tracking an inbound cruise missile. The operational parameters for the scenario are listed in Table 4.1. The sensor is mounted to a ship that is traveling at 10 kts on a NW course off the coast of South Korea. The sensor frame is directionally aligned with the vehicle (body) frame. The cruise missile target is traveling at 600 kts due East and, at the particular instant considered, is undergoing a 1-g acceleration in the East direction.

4.2 The Sensitivities

The transpose of the sensitivity matrix S is shown in Table 4.2. These sensitivities were computed using the simplifying assumptions that the sensor frame is fixed relative to the vehicle body frame and that the vehicle tilt (attitude) rates are sufficiently small. In addition, neither bearing rate nor elevation rate measurements are considered. The numerical entries in the table represent the amount of track velocity error in each of the three axes (x_e, y_e, z_e) of the WGS 84 coordinate frame that are generated by a unit error term. For example, if v_x is the first dimension of track velocity and vehicle east velocity is the fifth error, then a value of –0.777 for the (1,5) element of the sensitivity matrix

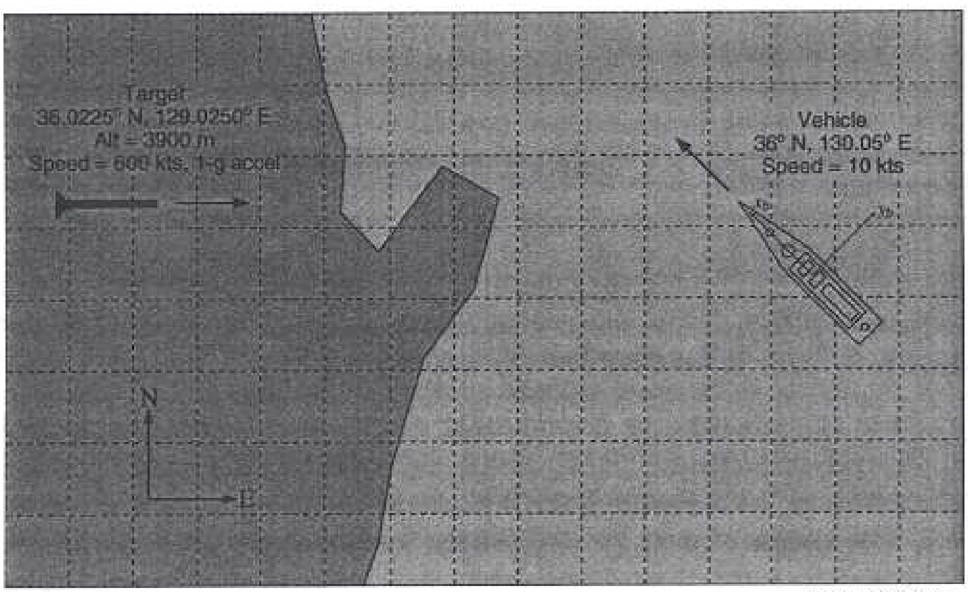

Figure 4.1: Example Tracking Scenario.

Item	Parameter	Value	Units
Vehicle	Latitude	deg N	36
	Longitude	deg E	130.05
	Altitude	m	0
	North velocity	kts	7.1
	East velocity	kts	–7.1
	Down velocity	m/s	0
	ϕ_b (roll)	deg	0
	θ_b (pitch)	deg	0
	ψ_b (yaw)	deg	315
	Lever arm, $(\Delta\vec{r}_{bs})_{b_1}$	m	–30
	Lever arm, $(\Delta\vec{r}_{bs})_{b_2}$	m	–8
	Lever arm, $(\Delta\vec{r}_{bs})_{b_3}$	m	–2
Sensor	ϕ_s (roll)	deg	0
	θ_s (pitch)	deg	0
	ψ_s (yaw)	deg	0
	Range to target	nm	50
	Range rate	m/s	–308
	Bearing to target	deg	317
	Elevation to target	deg	2
Network	Latency	sec	5
Aerospace Object	x_e-velocity	m/s	–240
	y_e-velocity	m/s	–194
	z_e-velocity	m/s	0
	x_e-acceleration	g	–0.78
	y_e-acceleration	g	–0.63
	z_e-acceleration	g	0

Table 4.1: Scenario Operational Parameters

means that every 1 m/s of east velocity error causes 0.777 m/s of track velocity error in the negative x_e-direction. If the east velocity error is 1 kt, then its contribution to track velocity error is 0.4 m/s in the negative x_e-direction.

Insight into how individual errors contribute to track velocity error can be gained by examining the root-sum-square (RSS) of the elements of the sensitivity matrix. These values, referred to as "Unit Radial Track Velocity Error," are provided in the last column of Table 4.2. The values of unity for Unit Radial Track Velocity Error corresponding to the three vehicle velocity error components imply that 1 m/s of vehicle velocity error in the north or east or down direction translates one-for-one into 1 m/s of track velocity error. This mapping is independent of operational conditions and scenario, *e.g.*, aerospace object conditions. The same holds for the contributions of sensor range rate error and tracker velocity errors.

Error Term	Unit	Sensitivities (m/s/unit) WGS 84 Earth Fixed Axes x_e	y_e	z_e	Unit Radial Track Velocity Error
North position	m	0.000	0.000	0.000	0.000
East position	m	0.000	0.000	0.000	0.000
Height	m	0.000	0.000	0.000	0.000
North velocity	m/s	0.379	−0.450	0.809	1.000
East velocity	m/s	−0.777	−0.630	0.000	1.000
Down velocity	m/s	−0.521	0.619	0.588	1.000
North tilt	mrad	0.113	−0.122	−0.130	0.211
East tilt	mrad	0.126	−0.138	−0.126	0.225
Down tilt	mrad	−0.109	0.145	−0.249	0.308
Lever arm, b_1	m	0.000	0.000	0.000	0.000
Lever arm, b_2	m	0.000	0.000	0.000	0.000
Lever arm, b_3	m	0.000	0.000	0.000	0.000
Sensor Misalignment, ψ_1	mrad	−0.113	0.122	0.130	0.211
Sensor Misalignment, ψ_2	mrad	−0.126	0.138	0.126	0.225
Sensor Misalignment, ψ_3	mrad	0.109	−0.145	0.249	0.308
Range to object	m	0.000	0.000	0.000	0.000
Range rate	m/s	0.759	0.650	0.047	1.000
Bearing to object	mrad	−0.109	0.145	−0.249	0.308
Elevation to object	mrad	0.169	−0.184	−0.181	0.308
Tracker x_e-position	m	0.000	0.000	0.000	0.000
Tracker y_e-position	m	0.000	0.000	0.000	0.000
Tracker z_e-position	m	0.000	0.000	0.000	0.000
Tracker x_e-velocity	m/s	1.000	0.000	0.000	1.000
Tracker y_e-velocity	m/s	0.000	1.000	0.000	1.000
Tracker z_e-velocity	m/s	0.000	0.000	1.000	1.000
Tracker x_e-acceleration	m/s^2	5.000	0.000	0.000	5.000
Tracker y_e-acceleration	m/s^2	0.000	5.000	0.000	5.000
Tracker z_e-acceleration	m/s^2	0.000	0.000	5.000	5.000
Timing[a]	sec	−7.629	−6.184	0.000	9.821

[a]Represents the sensitivity of any time error such as time synchronization, time stamp, etc.

Table 4.2: **Velocity** Error Sensitivity Matrix (Transposed) and Unit Radial Track Velocity Error

Track velocity error is not sensitive to position-type errors such as navigation position error, lever arm errors, sensor-measured range error, and tracker position errors. As already noted, track velocity error has a 1:1 sensitivity to velocity-type errors. Orientation-, or angular-type, errors contribute a small amount to track velocity error for this scenario. These include navigation tilt errors, sensor misalignments, and sensor-measured bearing and elevation errors. The sensitivity to navigation tilt errors is practically identical to the sensitivity to sensor misalignments due to the small lever arm vector. Likewise, the bearing and elevation sensitivities are effectively identical because

Error Term	Unit	Sensitivities (m/s/unit) WGS 84 Earth Fixed Axes			Unit Radial Track Error
		x_e	y_e	z_e	
North position	m	0.379	−0.450	0.809	1.000
East position	m	−0.777	−0.630	0.000	1.000
Height	m	−0.521	0.619	0.588	1.000
North velocity	m/s	0.000	0.000	0.000	0.000
East velocity	m/s	0.000	0.000	0.000	0.000
Down velocity	m/s	0.000	0.000	0.000	0.000
North tilt	mrad	−33.815	36.670	39.033	63.338
East tilt	mrad	−37.729	41.326	37.792	67.524
Down tilt	mrad	32.686	−43.490	74.769	92.468
Lever arm, b_1	m	0.809	0.137	0.572	1.000
Lever arm, b_2	m	−0.274	−0.773	0.572	1.000
Lever arm, b_3	m	0.521	−0.619	−0.588	1.000
Sensor Misalignment, ψ_1	mrad	33.812	−36.675	−39.027	63.336
Sensor Misalignment, ψ_2	mrad	37.738	−41.345	−37.814	67.553
Sensor Misalignment, ψ_3	mrad	−32.676	43.514	−74.785	92.488
Range to object	m	0.759	0.650	0.047	1.000
Range rate	m/s	0.000	0.000	0.000	0.000
Bearing to object	mrad	32.676	−43.514	74.785	92.488
Elevation to object	mrad	−50.657	55.247	54.279	92.544
Tracker x_e-position	m	1.000	0.000	0.000	1.000
Tracker y_e-position	m	0.000	1.000	0.000	1.000
Tracker z_e-position	m	0.000	0.000	1.000	1.000
Tracker x_e-velocity	m/s	5.000	0.000	0.000	5.000
Tracker y_e-velocity	m/s	0.000	5.000	0.000	5.000
Tracker z_e-velocity	m/s	0.000	0.000	5.000	5.000
Tracker x_e-acceleration	m/s^2	12.500	0.000	0.000	12.500
Tracker y_e-acceleration	m/s^2	0.000	12.500	0.000	12.500
Tracker z_e-acceleration	m/s^2	0.000	0.000	12.500	12.500
Timing[a]	sec	−277.937	−225.279	−0.005	357.770

[a]Represents the sensitivity of any time error such as time synchronization, time stamp, etc.

Table 4.3: **Position** Error Sensitivity Matrix (Transposed) and Unit Radial Track Error

of the low elevation of the target. The sensitivity to timing errors dominates the sensitivity matrix. This component is only dependent upon the acceleration of the aerospace object. The greater the acceleration, the stronger the sensitivity.

For comparison, the track position error sensitivity matrix is shown in Table 4.3. The first thing to note is that track position error is generally much more sensitive to the various error terms. For this scenario, angular-type errors and timing errors dominate the position error sensitivity matrix. There are some similarities between the velocity error sensitivities and the position error sensitivities. Specifically, we recognize that the position error sensitivities to navigation position errors and sensor-measured range errors are equivalent to the velocity error sensitivities to navigation velocity errors and sensor-measured range rate errors, which are just the time rates of change of navigation position and range.

4.3 System Error Values

A nominal error budget for this scenario is presented in Table 4.4. The errors are assumed to be normally distributed with zero mean. For this analysis, all errors are assumed to be independent. However, the methodology presented in Section 2 does not require this to be true. The uncorrelated model was chosen for convenience. A detailed analysis of the individual subsystems is necessary to develop the more general correlated error model.

4.4 Track Velocity Error Budget

Combining the sensitivity matrix in Table 4.2 with the nominal error budget in Table 4.4 leads to the nominal track velocity accuracy error budget displayed in Figure 4.2. The bars in the figure are color-coded to help visualize the roll-up of individual errors. Navigation errors are colored in light blue, sensor errors are medium blue, tracker errors are green, and timing errors are [illegible]. Above the horizontal dotted line are the individual error contributors and below the line are the groupings of error contributors, which are color-coded accordingly. In addition, the data registration error roll-up (orange), the dynamic error roll-up (red), and the total system error (**black**) are shown.

The track velocity accuracy is quantified in terms of the 95% radius, which represents the 'radius' of a sphere that encompasses 95% of the track velocity error distribution. Unlike the case of track position error, this 95% sphere does not have a physical interpretation since the 'radius' is in units of m/s. Referring to Figure 4.2, the length of the bar for each error term listed in the chart reflects the contribution to track velocity error, expressed as 95% radius, if that error term was the only error in the system. For instance, if range rate were the only error, the 95% radius for the system would be about 20 m/s.

This chart also illustrates the cumulative effects of several errors. For example, the sensor range, range rate, bearing, and elevation error contributors combine, or roll up, to give a total MEASUREMENT error contribution of approximately 20 m/s 95% radius, which is dominated by the range rate error. Similarly, the tracker position, velocity, and acceleration error contributors combine to yield a total TRACKER error contribution of approximately 50 m/s 95% radius. The total system error of 53.6 m/s 95% radius is composed of a DATA REGSTRN (NAVIGATION + LEVER ARM + SENSOR) contribution of 19.4 m/s 95% radius and a DYNAMIC (TRACKER + TIME) contribution of 50.9 m/s 95% radius. For comparison, the total system error for track position is 781.3 m 95% radius for this scenario. Consequently, it would take almost 15 seconds for this velocity error (53.6 m/s) to grow into an equivalent position error of 781 m. This relatively

Item	Error Parameter	Unit	1-Sigma Value
Vehicle	North position	m	200
	East position	m	200
	Height	m	20
	North velocity	m/s	0.25
	East velocity	m/s	0.25
	Down velocity	m/s	0.10
	North tilt	mrad	0.50
	East tilt	mrad	0.50
	Down tilt	mrad	0.50
	Lever arm, $(\delta\vec{r}_{bs})_{b_1}$	m	0.10
	Lever arm, $(\delta\vec{r}_{bs})_{b_2}$	m	0.10
	Lever arm, $(\delta\vec{r}_{bs})_{b_3}$	m	0.10
Sensor	Sensor Misalignment, $\delta\psi_1$	mrad	1
	Sensor Misalignment, $\delta\psi_2$	mrad	1
	Sensor Misalignment, $\delta\psi_3$	mrad	1
	Range to object	m	100
	Range rate	m/s	10
	Bearing to object	mrad	2
	Elevation to object	mrad	2
Tracker	Tracker x_e-position	m	10
	Tracker y_e-position	m	10
	Tracker z_e-position	m	10
	Tracker x_e-velocity	m/s	15
	Tracker y_e-velocity	m/s	15
	Tracker z_e-velocity	m/s	15
	Tracker x_e-acceleration	m/s^2	2
	Tracker y_e-acceleration	m/s^2	2
	Tracker z_e-acceleration	m/s^2	2
Timing	Host-to-host	sec	0.25
	Host-to-network	sec	0.25
	Time stamp	sec	0.25
	Time latency	sec	0.25

Table 4.4: Nominal Scenario Error Budget

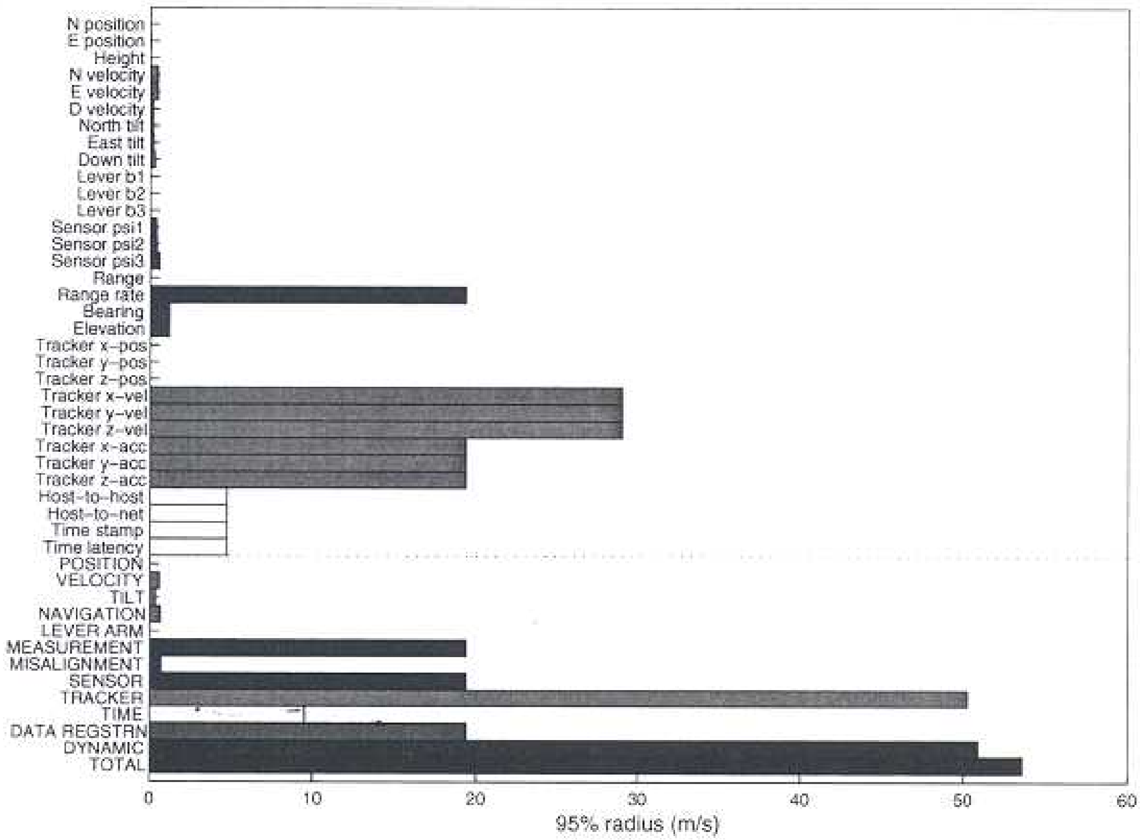

Figure 4.2: Nominal Track Velocity Accuracy Error Budget.

long time implies that track velocity error is not as important as track position error for successful aerospace object tracking.

The error budget results provide an easy way to identify the major contributors to total track velocity error, at both the subsystem level and at the individual error component level. For this scenario, dynamic errors dominate the total system error. These dynamic errors are composed of tracker and timing error contributions. Specifically, the tracker velocity and acceleration error contributions are the primary cause for the large tracker error contribution. Therefore, improvements to the tracker algorithms would yield the greatest benefit in terms of reducing track velocity error. Additional benefit could be gained with an improved sensor measurement of range rate. Improvements to the vehicle navigation system would have little effect on track velocity error as evidenced by their negligible contributions in the chart.

5 CONCLUSIONS

This Technical Report has provided the details of an analytical technique to understand and quantify how errors in navigation, sensor, timing, and tracker affect the accuracy of track velocity estimation. The analysis approach derives the equation for the velocity of an aerospace object in the WGS 84 coordinate frame and determines how data registration and dynamic errors affect the estimate of track velocity. An examination of the components making up the track velocity vector reveals three groups of error contributors:

- Vehicle position, velocity, and attitude
- Sensor measurements and misalignments
- Timing (*e.g.*, time stamp, time synchronization, time latency, etc.)
- Tracker

Comparing these results for velocity error to those for position error, we identify new contributors to track velocity error:

- Vehicle velocity
- Sensor rate measurements (if available)

Vehicle tilt rate (attitude rate) errors do not enter into the track velocity error in this analysis because of the simplifying assumptions made in Section 2. After the error mechanisms were identified, a first-order linear perturbation analysis was employed to develop the relationship between track velocity error and the errors identified above.

A key result of the analysis is the determination of the sensitivities of track velocity accuracy to the navigation, sensor, timing, and tracker errors. With the sensitivities established, a nominal tracking scenario was considered and a detailed track velocity error budget generated. The error budget results provided an easy way to identify the major contributors to total track velocity error, at both the subsystem and individual error component level. This insight shows where improvements in the system could be made in order to yield the greatest benefit in terms of track velocity error. The results further indicate that track velocity error contributors are less important than track position error contributors for successful aerospace object tracking. As a result, the Integrated Architecture Behavior Model (IABM) development activities should focus on capturing the position error contributors for the Configuration 05 build.

Finally, the results from this analysis can be used to investigate track velocity error and will be incorporated into the JSSEO Parametric Track Accuracy Model thereby enhancing its scope and capability. For example, we are using the error model and the results of the analysis runs to develop test cases to assist with the IABM verification and validation effort. The error models provide the mechanism to introduce the correct level of bias into each system, and the results of the error model analysis provide examples of track velocity error that the IABM can be validated against.

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APPENDIX A: SUMMARY OF POSITION ERROR ANALYSIS RESULTS

A.1 The Earth

We consider the WGS 84 ellipsoidal model of the Earth geoid (see Table B.1). The geodetic coordinates of a point B with respect to this datum will be denoted as (ϕ, λ, h), which are geodetic latitude, longitude, and altitude of point B above the ellipsoid. The Conventional Terrestrial System (CTS) associated with the WGS 84 datum will form the right-handed Cartesian Earth frame, or e-frame.

A.2 The Vehicle

We consider a vehicle, fixed or moving, in the vicinity of the Earth. We define a point B, fixed to the vehicle, representing the center of the vehicle's Inertial Navigation System (INS) with geodetic coordinates (ϕ, λ, h) as above. The vector from the center of the Earth to point B on the vehicle, expressed in the e-frame, is given by

$$(\vec{r}_b)_e = \begin{bmatrix} (r_{ew} + h)C_\phi C_\lambda \\ (r_{ew} + h)C_\phi S_\lambda \\ [(1 - e^2)r_{ew} + h]\, S_\phi \end{bmatrix}_e , \tag{A.1}$$

where C_α and S_α are shorthand for $\cos\alpha$ and $\sin\alpha$, and r_{ew} is the east–west radius of curvature given by

$$r_{ew} = a_e(1 - e^2 S_\phi^2)^{-1/2}. \tag{A.2}$$

A.3 The Aerospace Object Position Vector Processing Chain

The following is a summary of the nominal (error-free) processing chain for the aerospace object position vector. The starting point are the sensor measurements of the object and the ending point is the object position vector, which is the vector from the center of the Earth to the aerospace object, expressed in Earth coordinates.

Step 1: Assume that the sensor system associated with the vehicle measures the range ρ, the bearing ψ_s, and the elevation θ_s of the object with respect to the sensor frame (s-frame). This provides the sensor–object displacement vector in sensor coordinates:

$$(\Delta\vec{r}_{st})_s = \rho \begin{bmatrix} C_{\theta_s} C_{\psi_s} \\ C_{\theta_s} S_{\psi_s} \\ -S_{\theta_s} \end{bmatrix}_s . \tag{A.3}$$

Step 2: The sensor–object displacement vector is transformed to body coordinates:

$$(\Delta\vec{r}_{st})_b = R_{bs}(\Delta\vec{r}_{st})_s. \tag{A.4}$$

The sensor–body rotation matrix, R_{bs}, is given by

$$R_{bs} = R_1(\phi_s)R_2(\theta_s)R_3(\psi_s). \tag{A.5}$$

where $(\psi_s, \theta_s, \phi_s)$ are the three Euler angles describing the rotation from the sensor frame to the body frame, and R_1, R_2, and R_3 are the standard trio of single-axis rotation matrices defined as

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix},$$

$$R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \tag{A.6}$$

$$R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Step 3: The body–sensor lever arm, $(\Delta\vec{r}_{bs})_b$, is added to produce the body–object displacement vector in body coordinates:

$$(\Delta\vec{r}_{bt})_b = (\Delta\vec{r}_{bs})_b + (\Delta\vec{r}_{st})_b. \tag{A.7}$$

Step 4: The body–object displacement vector is transformed to local-level coordinates:

$$(\Delta\vec{r}_{bt})_l = R_{bl}^T(\Delta\vec{r}_{bt})_b. \tag{A.8}$$

The local-level–body rotation matrix, R_{bl}, is given by

$$R_{bl} = R_1(\phi_b)R_2(\theta_b)R_3(\psi_b), \tag{A.9}$$

where $(\psi_b, \theta_b, \phi_b)$ are the three Euler angles describing the rotation from the local-level frame to the body frame.

Step 5: The body–object displacement vector is transformed to Earth coordinates:

$$(\Delta\vec{r}_{bt})_e = R_{le}^T(\Delta\vec{r}_{bt})_l. \tag{A.10}$$

The Earth–local-level rotation matrix, R_{le}, is given by

$$R_{le} = R_2\left(-\frac{\pi}{2} - \phi\right) R_3(\lambda) = \begin{bmatrix} -S_\phi C_\lambda & -S_\phi S_\lambda & C_\phi \\ -S_\lambda & C_\lambda & 0 \\ -C_\phi C_\lambda & -C_\phi S_\lambda & -S_\phi \end{bmatrix}. \tag{A.11}$$

Step 6: The navigation system position vector, $(\vec{r}_b)_e$, is added to produce the object position vector in WGS 84 coordinates:

$$(\vec{r}_t)_e = (\vec{r}_b)_e + (\vec{r}_{bt})_e. \tag{A.12}$$

A.4 The Position Vector Chain of Errors

The same steps as in the preceding section are used to derive the measured quantities from the fundamental error-free quantities. Measured quantities will be denoted with a $\tilde{}$ and error quantities will be denoted with a preceding δ.

Step 1: Error in the sensor–object displacement vector in sensor coordinates:

$$(\Delta\tilde{\vec{r}}_{st})_s = (\Delta\vec{r}_{st})_s + (\delta\vec{r}_{st})_s, \tag{A.13}$$

$$(\delta\vec{r}_{st})_s = -\rho\delta\theta_s\begin{bmatrix} S_{\theta_s}C_{\psi_s} \\ S_{\theta_s}S_{\psi_s} \\ C_{\theta_s} \end{bmatrix}_s + \rho\delta\psi_s\begin{bmatrix} -C_{\theta_s}S_{\psi_s} \\ C_{\theta_s}C_{\psi_s} \\ 0 \end{bmatrix}_s + \delta\rho\begin{bmatrix} C_{\psi_s}C_{\theta_s} \\ S_{\psi_s}C_{\theta_s} \\ -S_{\theta_s} \end{bmatrix}_s. \tag{A.14}$$

Step 2: Error in the sensor–object displacement vector in body coordinates:

$$(\Delta\tilde{\vec{r}}_{st})_b = (\Delta\vec{r}_{st})_b + (\delta\vec{r}_{st})_b, \tag{A.15}$$

$$(\delta\vec{r}_{st})_b = R_{bs}(\delta\vec{r}_{st})_s + \delta R_{bs}(\Delta\vec{r}_{st})_s, \tag{A.16}$$

$$\delta R_{bs} = \delta\Omega_{bs}\tilde{R}_{bs}, \tag{A.17}$$

$$\begin{bmatrix} 0 & \delta\psi_3 & -\delta\psi_2 \\ -\delta\psi_3 & 0 & \delta\psi_1 \\ \delta\psi_2 & -\delta\psi_1 & 0 \end{bmatrix} = \delta\Omega_{bs} \Leftrightarrow (\delta\vec{\psi})_b = \begin{bmatrix} \delta\psi_1 \\ \delta\psi_2 \\ \delta\psi_3 \end{bmatrix}_b. \tag{A.18}$$

Step 3: Error in the body–object displacement vector in body coordinates:

$$(\Delta\tilde{\vec{r}}_{bt})_b = (\Delta\vec{r}_{bt})_b + (\delta\vec{r}_{bt})_b, \tag{A.19}$$

$$(\delta\vec{r}_{bt})_b = (\delta\vec{r}_{bs})_b + (\delta\vec{r}_{st})_b. \tag{A.20}$$

Step 4: Error in the body–object displacement vector in local-level coordinates:

$$(\Delta\tilde{\vec{r}}_{bt})_l = (\Delta\vec{r}_{bt})_l + (\delta\vec{r}_{bt})_l, \tag{A.21}$$

$$(\delta\vec{r}_{bt})_l = R_{bl}^T(\delta\vec{r}_{bt})_b + \delta R_{bl}^T(\Delta\vec{r}_{bt})_b, \tag{A.22}$$

$$\delta R_{bl} = \delta\Omega_{bl}\tilde{R}_{bl}, \tag{A.23}$$

$$\delta\Omega_{bl} = \begin{bmatrix} 0 & \delta\theta_D & -\delta\theta_E \\ -\delta\theta_D & 0 & \delta\theta_N \\ \delta\theta_E & -\delta\theta_N & 0 \end{bmatrix}, \tag{A.24}$$

$$(\delta\vec{\theta})_b = \begin{bmatrix} \delta\theta_N \\ \delta\theta_E \\ \delta\theta_D \end{bmatrix}_b = \delta\phi_b\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_b + \delta\theta_b\begin{bmatrix} 0 \\ C_{\phi_b} \\ -S_{\phi_b} \end{bmatrix}_b + \delta\psi_b\begin{bmatrix} -S_{\theta_b} \\ C_{\theta_b}S_{\phi_b} \\ C_{\theta_b}C_{\phi_b} \end{bmatrix}_b. \tag{A.25}$$

Step 5: Error in the body–object displacement vector in Earth coordinates:

$$(\Delta\widetilde{\vec{r}}_{bt})_e = (\Delta\vec{r}_{bt})_e + (\delta\vec{r}_{bt})_e, \tag{A.26}$$

$$(\delta\vec{r}_{bt})_e = R_{le}^T(\delta\vec{r}_{bt})_l + \delta R_{le}^T(\Delta\vec{r}_{bt})_l, \tag{A.27}$$

$$\delta R_{le} = \delta\Omega_{le}\widetilde{R}_{le}, \tag{A.28}$$

$$\delta\Omega_{le} = \begin{bmatrix} 0 & \delta\phi_3 & -\delta\phi_2 \\ -\delta\phi_3 & 0 & \delta\phi_1 \\ \delta\phi_2 & -\delta\phi_1 & 0 \end{bmatrix}, \tag{A.29}$$

$$(\delta\vec{\phi})_l = \begin{bmatrix} \delta\phi_1 \\ \delta\phi_2 \\ \delta\phi_3 \end{bmatrix}_l = -\delta\phi\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_l + \delta\lambda\begin{bmatrix} C_\phi \\ 0 \\ -S_\phi \end{bmatrix}_l. \tag{A.30}$$

Step 6: Error in the aerospace object position vector in Earth coordinates:

$$(\widetilde{\vec{r}}_t)_e = (\vec{r}_t)_e + (\delta\vec{r}_t)_e, \tag{A.31}$$

$$(\delta\vec{r}_t)_e = (\delta\vec{r}_b)_e + (\delta\vec{r}_{bt})_e, \tag{A.32}$$

$$(\delta\vec{r}_b)_e = (r_{ns}+h)\delta\phi\begin{bmatrix} -S_\phi C_\lambda \\ -S_\phi S_\lambda \\ C_\phi \end{bmatrix}_e + (r_{ew}+h)C_\phi\delta\lambda\begin{bmatrix} -S_\lambda \\ C_\lambda \\ 0 \end{bmatrix}_e + \delta h\begin{bmatrix} C_\phi C_\lambda \\ C_\phi S_\lambda \\ S_\phi \end{bmatrix}_e, \tag{A.33}$$

where r_{ns} is the north–south radius of curvature given by

$$r_{ns} = a_e(1-e^2)(1-e^2S_\phi^2)^{-3/2} = r_{ew}(1-e^2)(1-e^2S_\phi^2)^{-1}. \tag{A.34}$$

APPENDIX B: WGS 84 ELLIPSOID PARAMETERS

Consider an ellipsoid model of the Earth geoid, in particular, the one used as the geodetic datum by the World Geodetic System 1984 (WGS 84). Parameters related to this ellipsoid are listed in Table B.1.

Parameter	Notation	Formula	Value
Angular velocity	ω_e		$7.2921151467 \times 10^{-5}$ rad/sec 15.04106718 deg/hr
Semimajor axis	a_e		6378137 m
Flattening	f		1/298.2572236
Semiminor axis	b_e	$a_e(1-f)$	6356752.3 m
Eccentricity	e	$\sqrt{f(2-f)}$	0.08181919084
Eccentricity squared	e^2	e^2	0.00669437990
Axis ratio	b_e/a_e	$\sqrt{1-e^2}$	0.9966471893
Axis ratio squared	$(b_e/a_e)^2$	$1-e^2$	0.9933056199
Linear eccentricity	E	ea_e	521854.01 m
Minor eccentricity	e'	$e/(1-f)$	0.08209443796

Table B.1: WGS 84 ellipsoid parameters